The Round Complexity of Secure Computation Against Covert Adversaries

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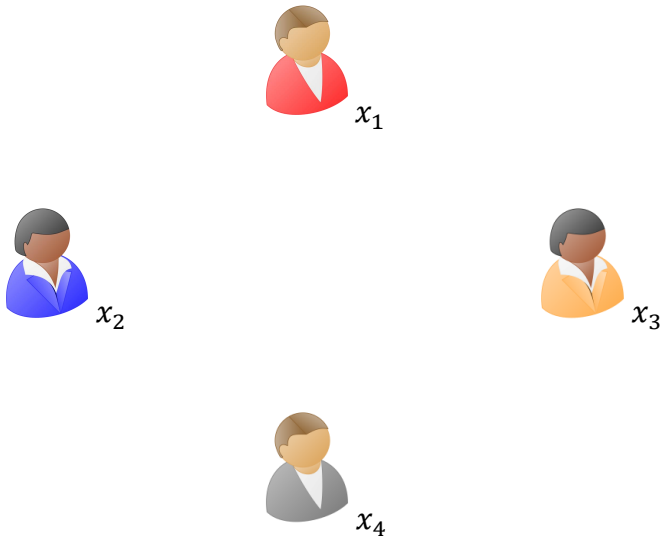
Johns Hopkins University

SCN 2020

Multiparty Computation (MPC)

[Yao'86, Goldreich-Micali-Wigderson'87]



$y = f(x_1, x_2, x_3, x_4)$

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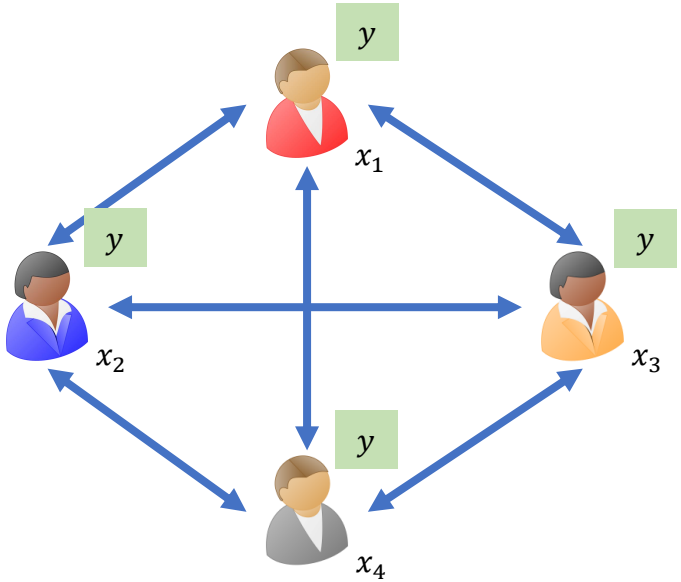

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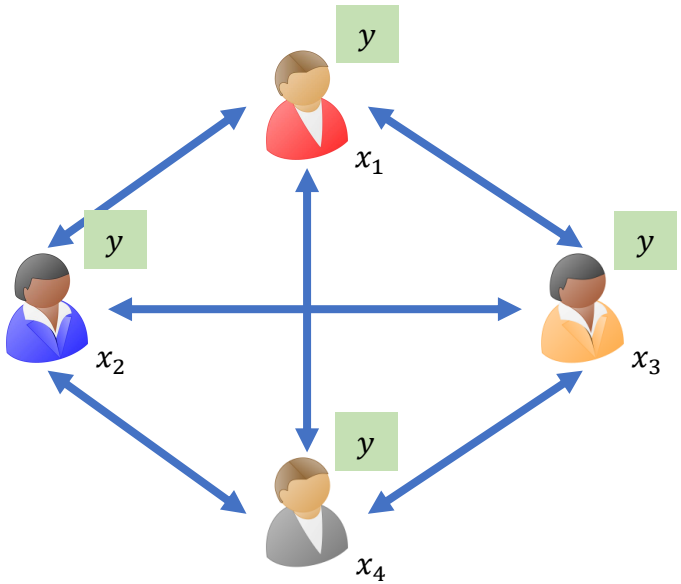

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A **round** constitutes of every participant sending a message.

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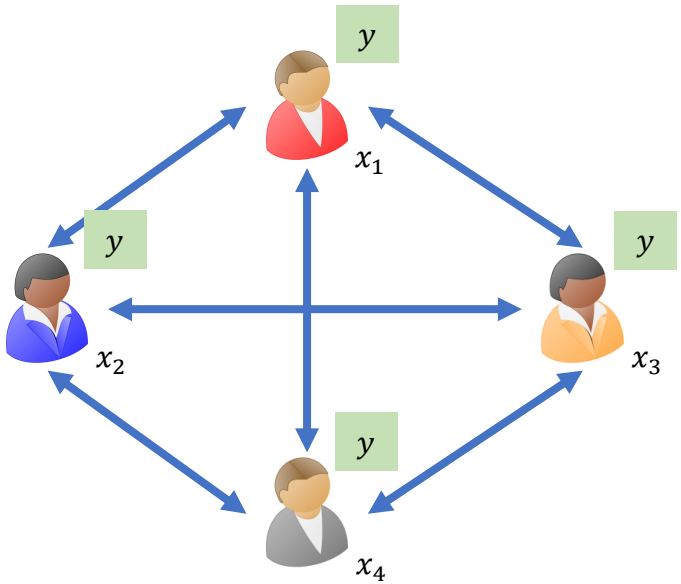

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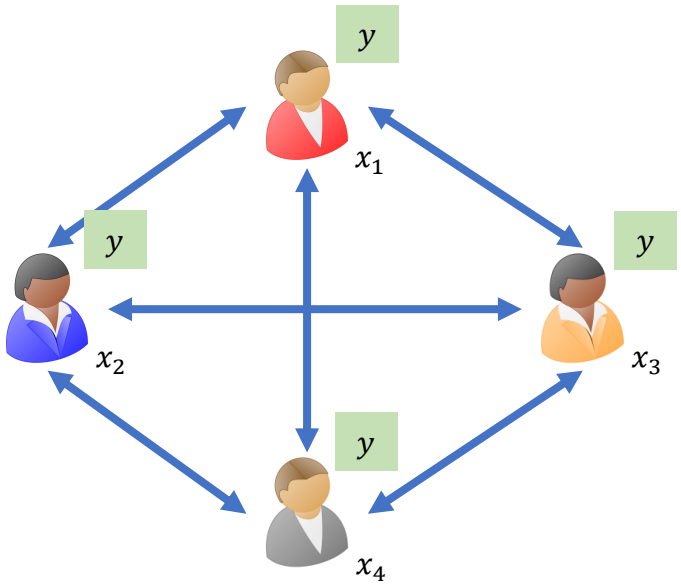
A **round** constitutes of every participant sending a message.

Goal: For efficiency, **minimize rounds of interaction**.

Security



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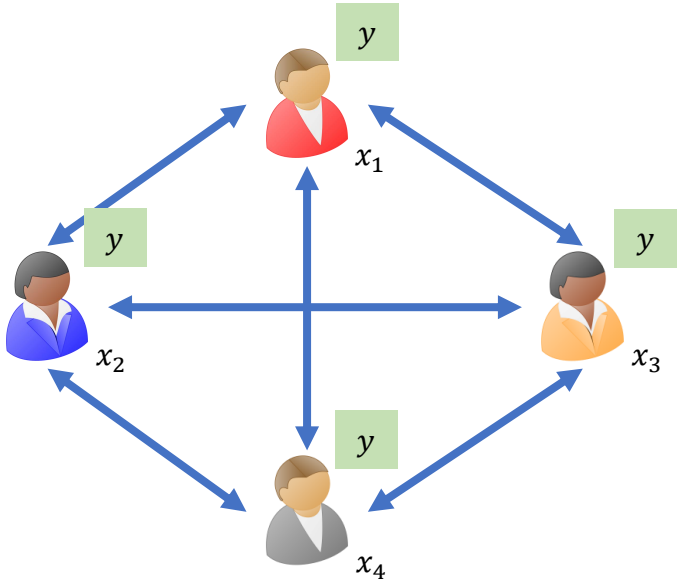
Misbehaving participants should not learn anything beyond the output of the function.

Security



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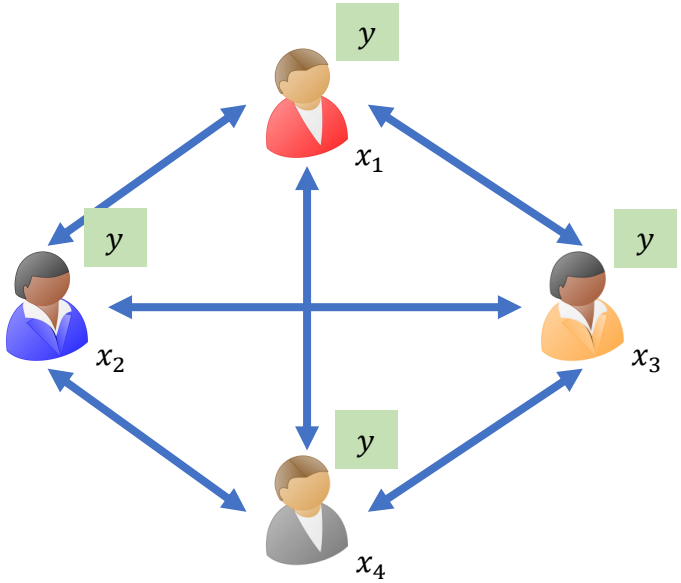
Security

real world



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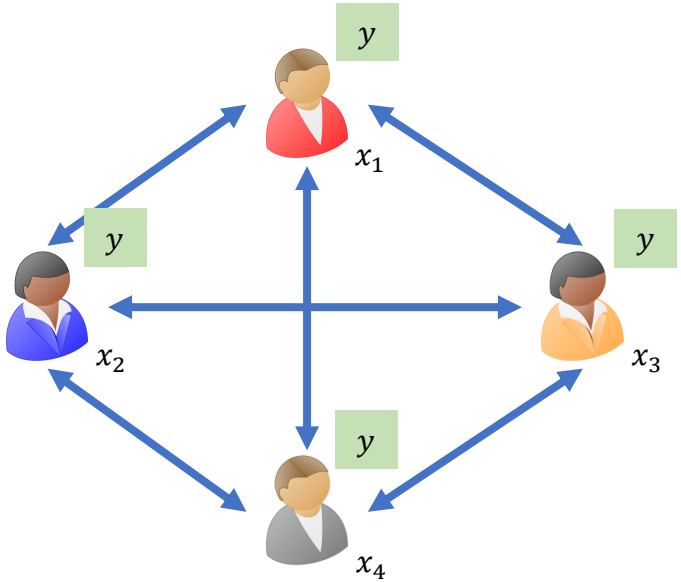

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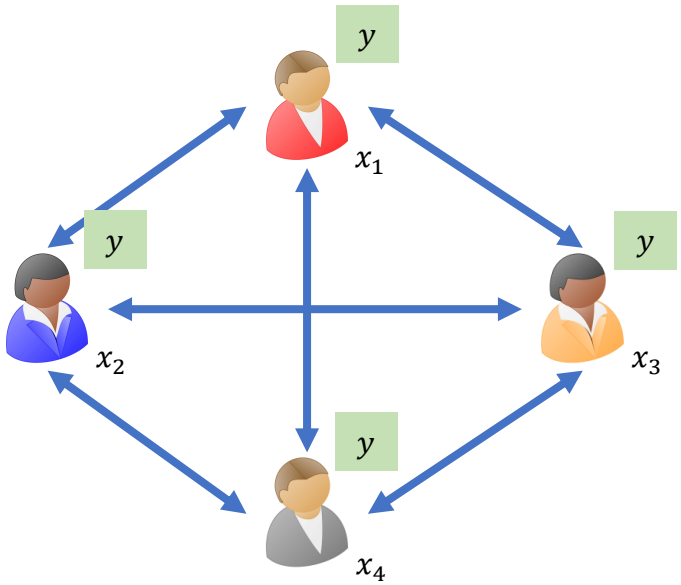
ideal world

Security

real world



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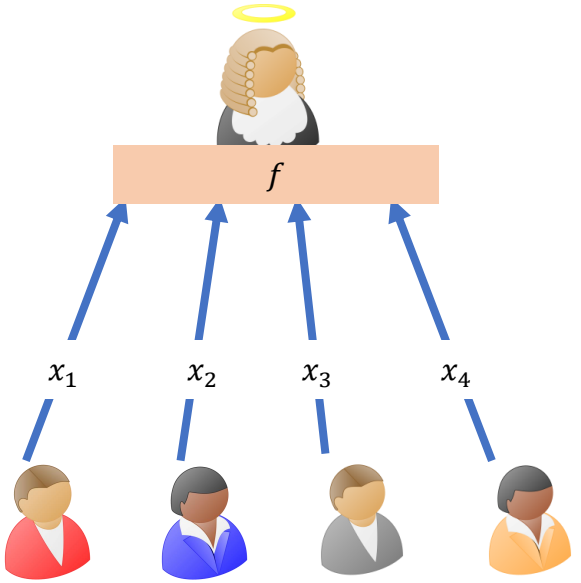

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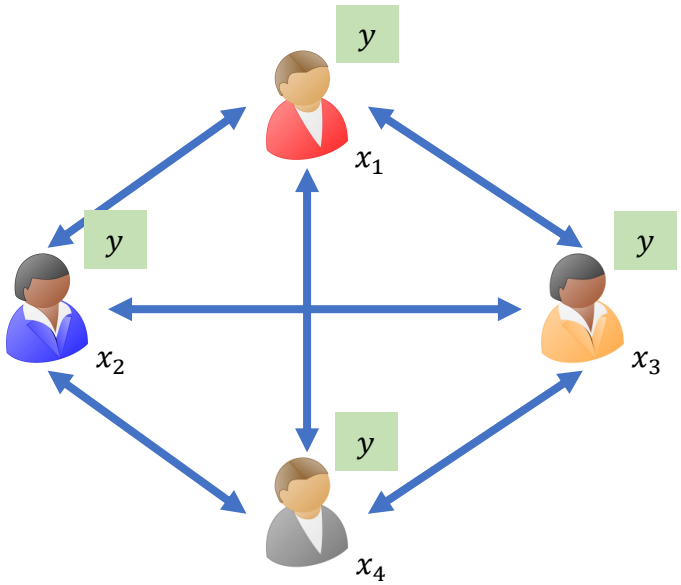
real world



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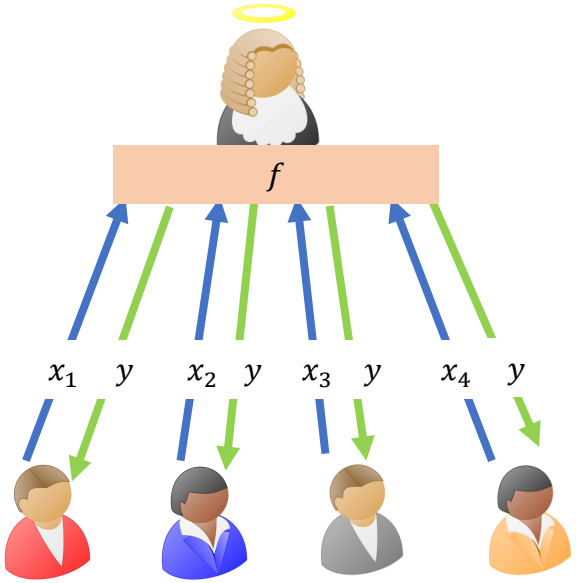

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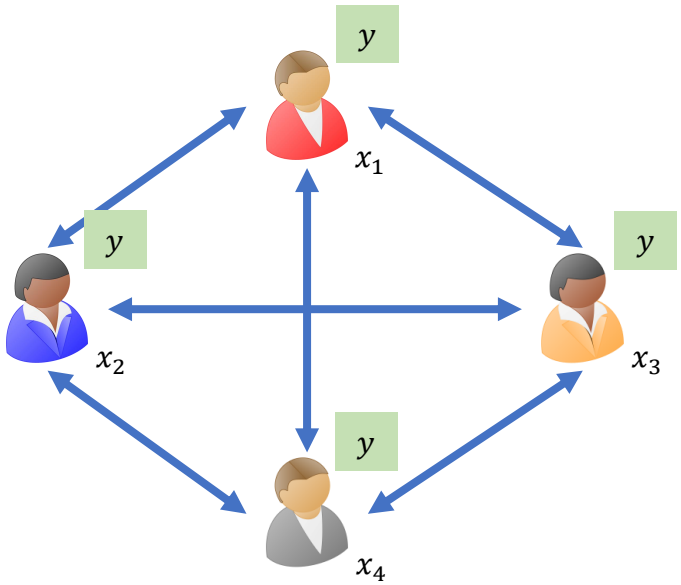

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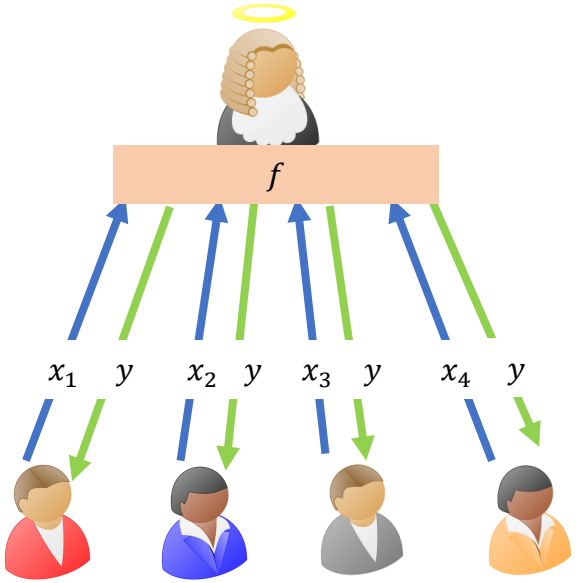

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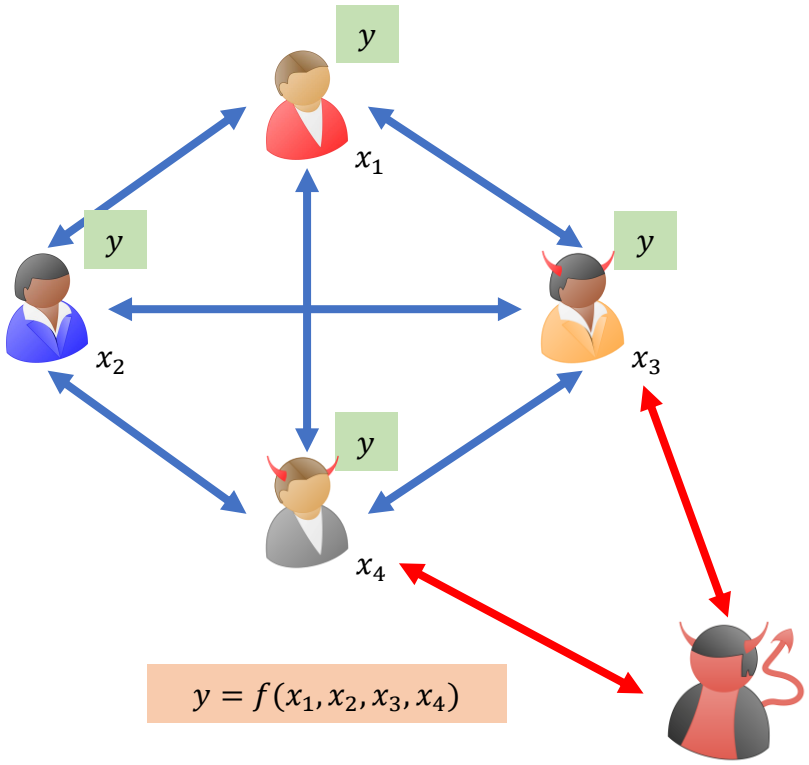
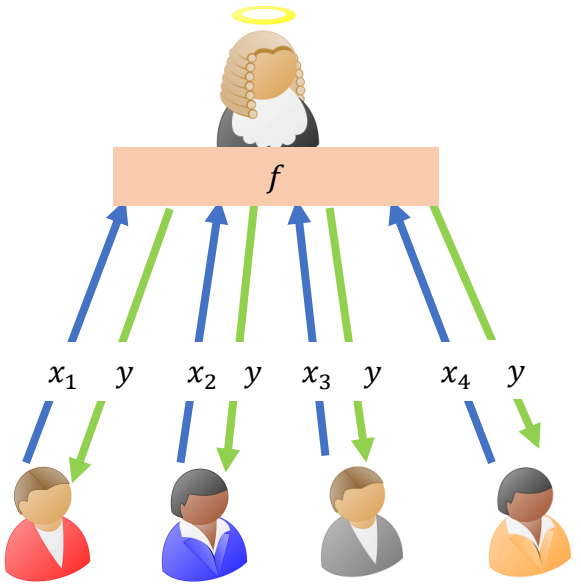
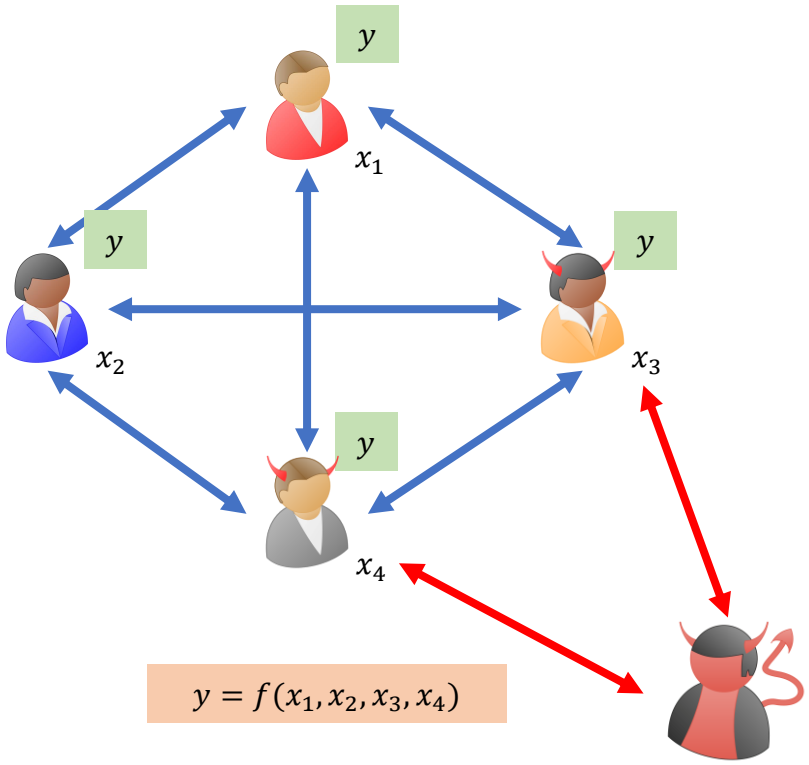


ideal world



Security

real world



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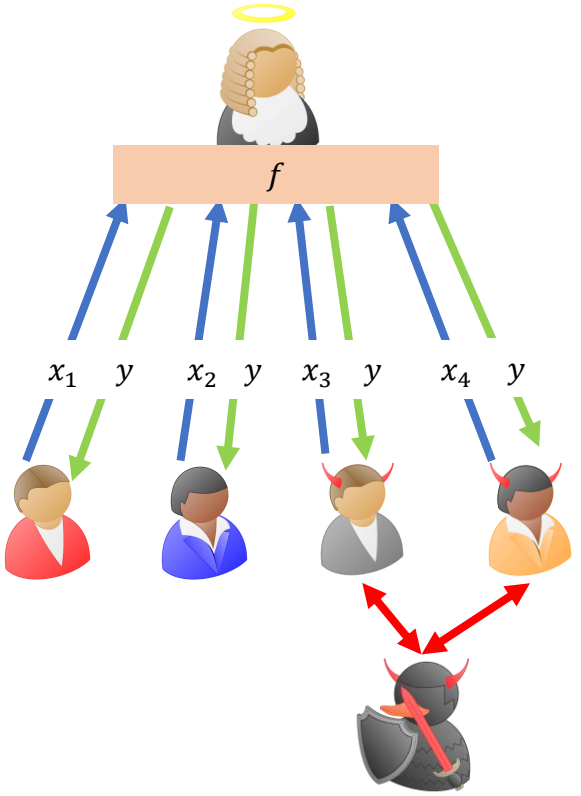

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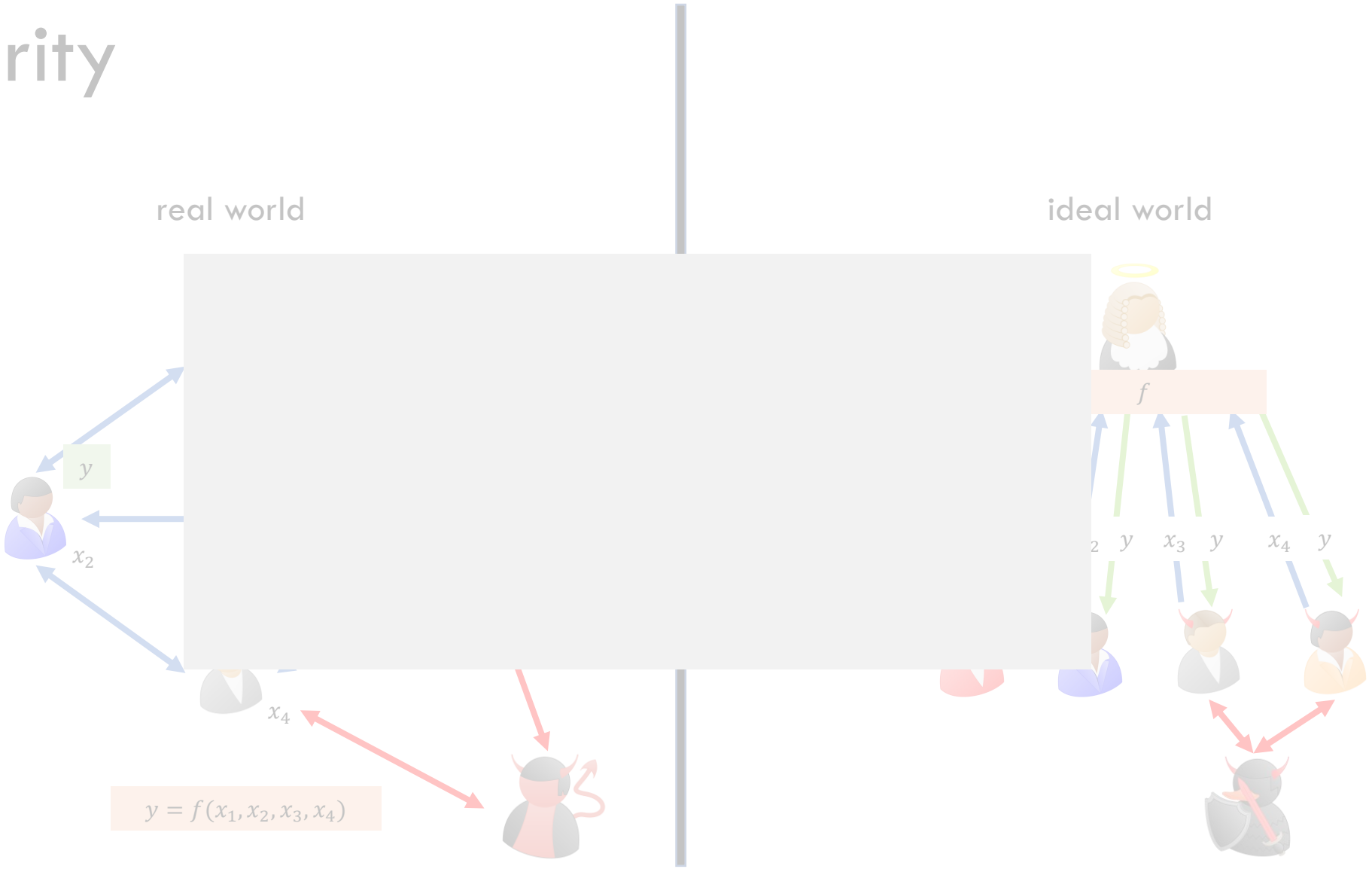

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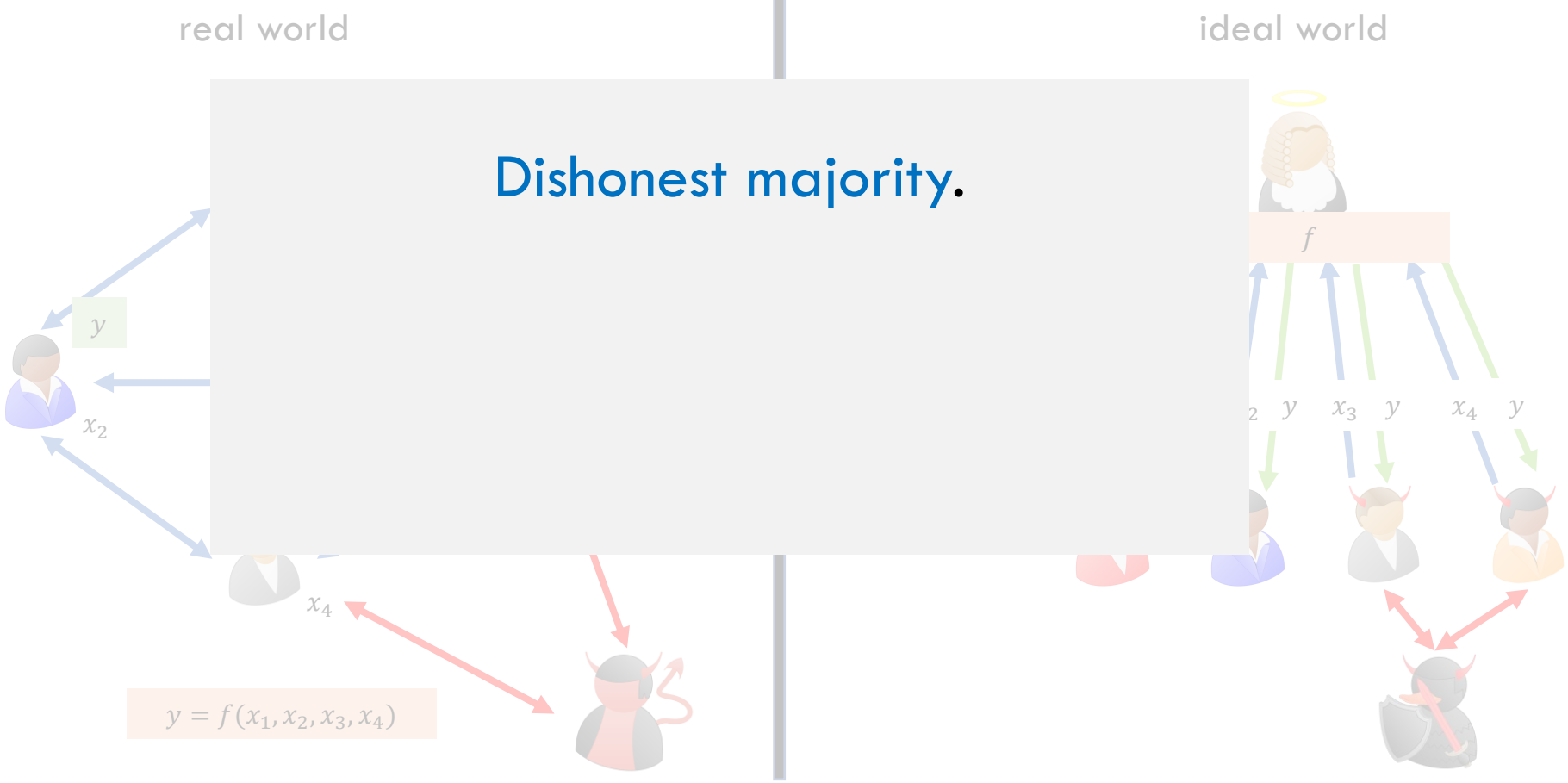
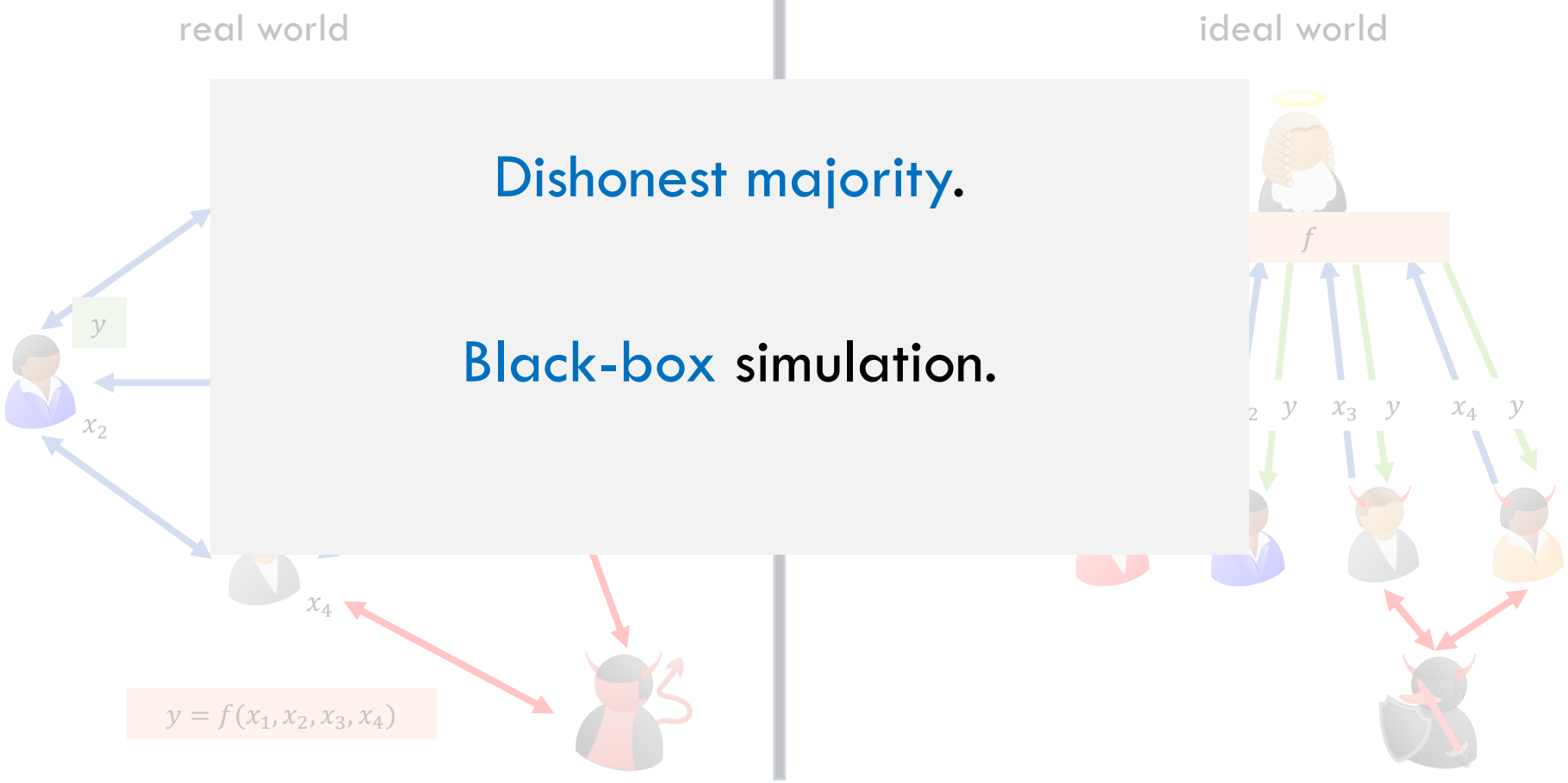

real world
ideal world
Dishonest majority.
y
x_2
x_4
$y = f(x_1, x_2, x_3, x_4)$
f
y
x_3
y
x_4
y

Security


real world
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Dishonest majority.
Black-box simulation.
y
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Known Round Complexity Bounds
(Semi-Honest)

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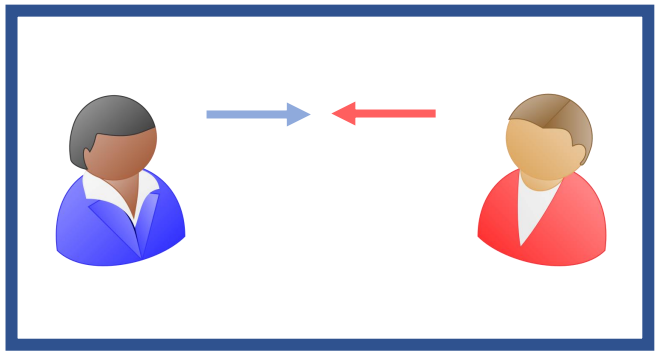

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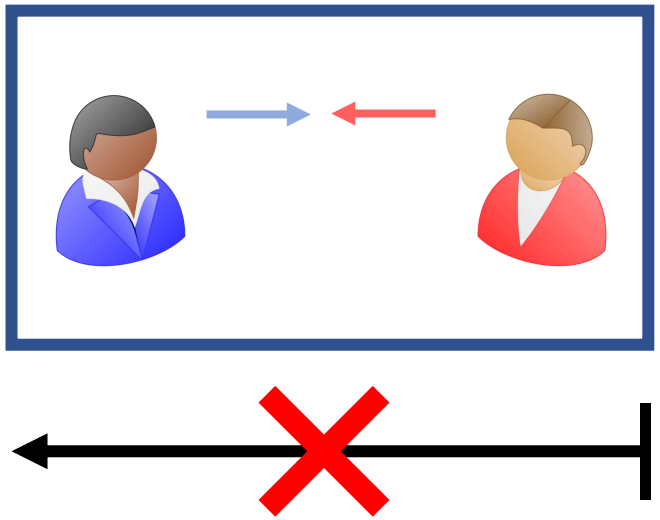

Impossible

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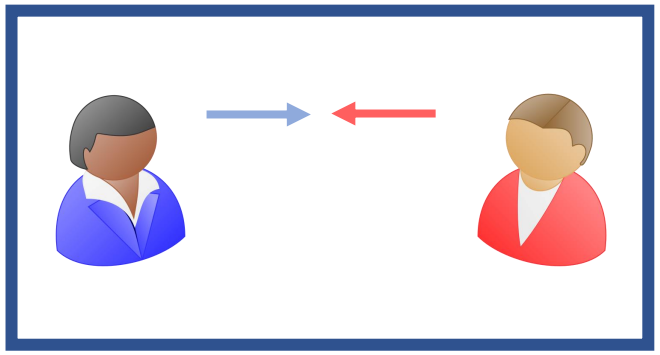

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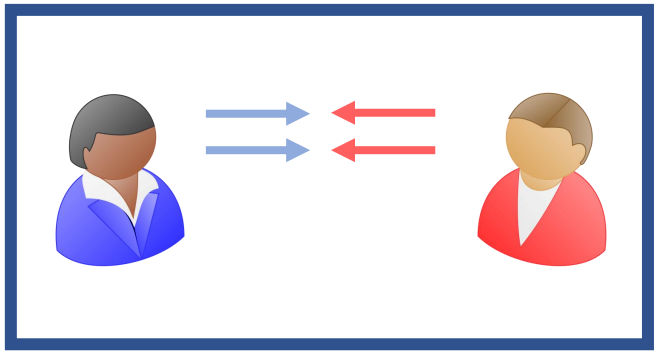

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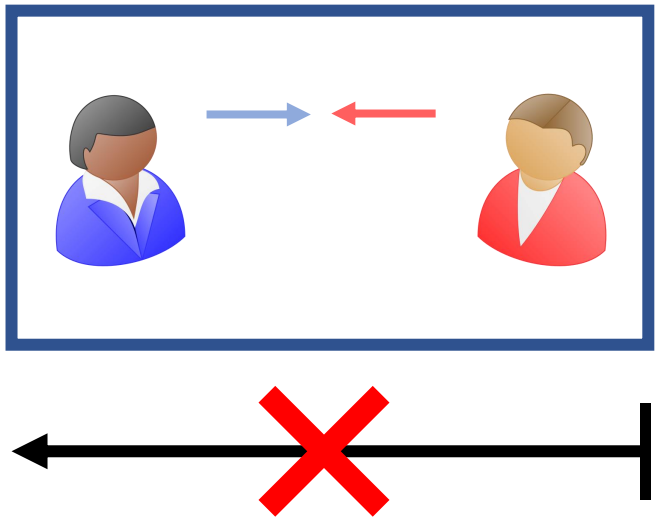

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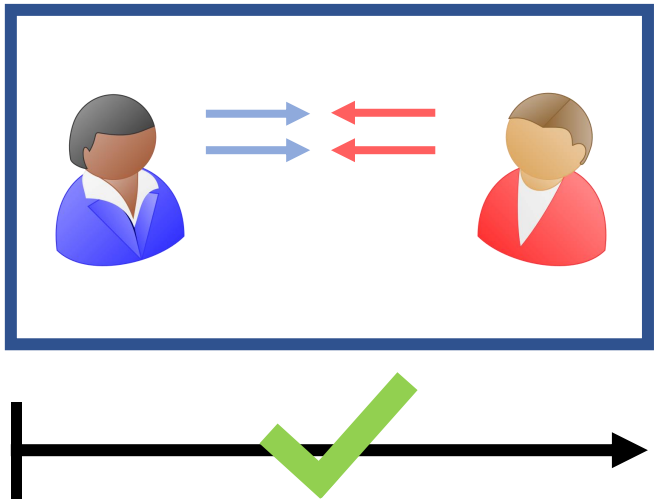

Possible

[Garg-Gentry-Halevi-Raykova'14]
[Mukherjee-Wichs'16]
[Garg-Srinivasan'17]
[Garg-Srinivasan'18]
[Benhamouda-Lin'18]

Known Round Complexity Bounds
(Malicious)

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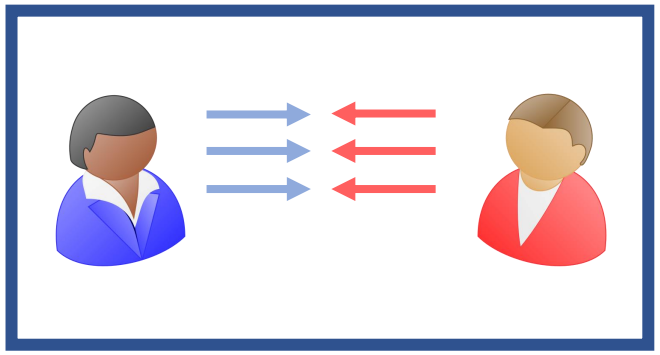

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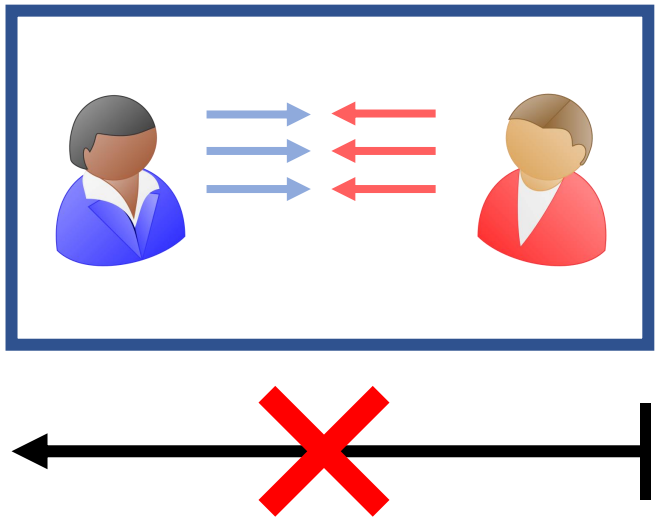

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[Garg-Mukherjee-Pandey-Polychroniadou'16]

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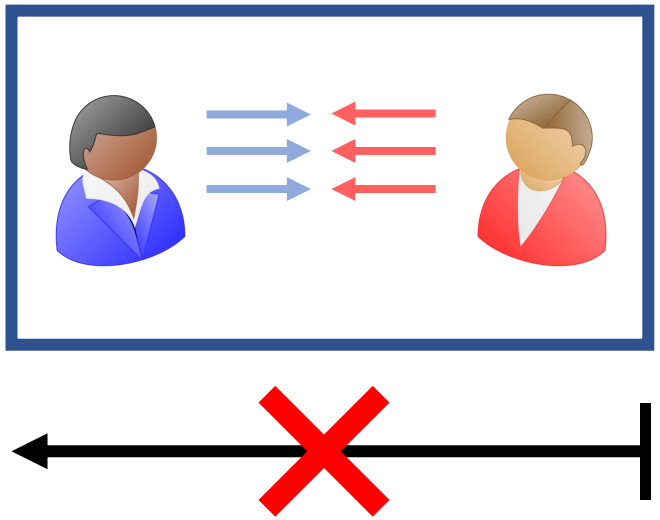

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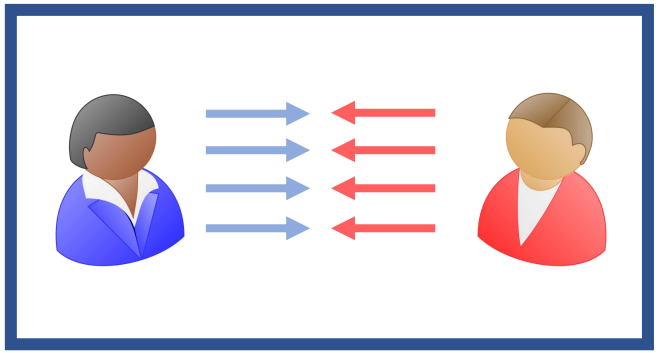

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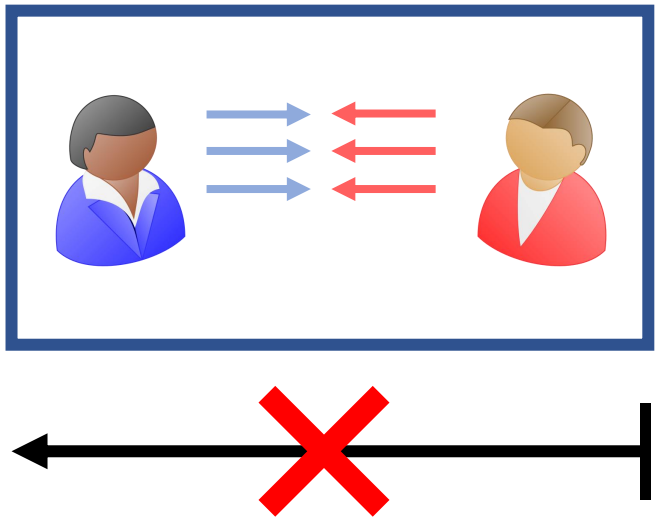

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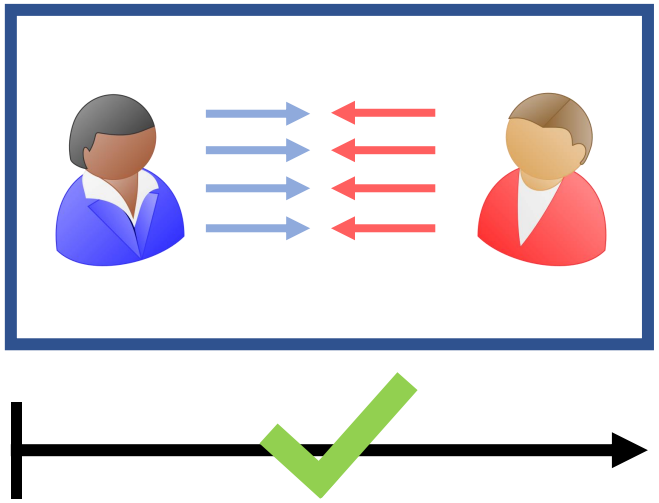

Possible

[Ananth-**C**-Jain'17]
[Brakerski-Halevi-Polychroniadou'17]
[Badrinarayanan-Goyal-Jain-Kalai-Khurana-Sahai'18]
[Halevi-Hazay-Polychroniadou-Venkitasubramaniam18]
[**C**-Ciampi-Goyal-Jain-Ostrovsky'19]

Can we design a protocol that spans the spectrum of round complexity with an intermediate security notion?

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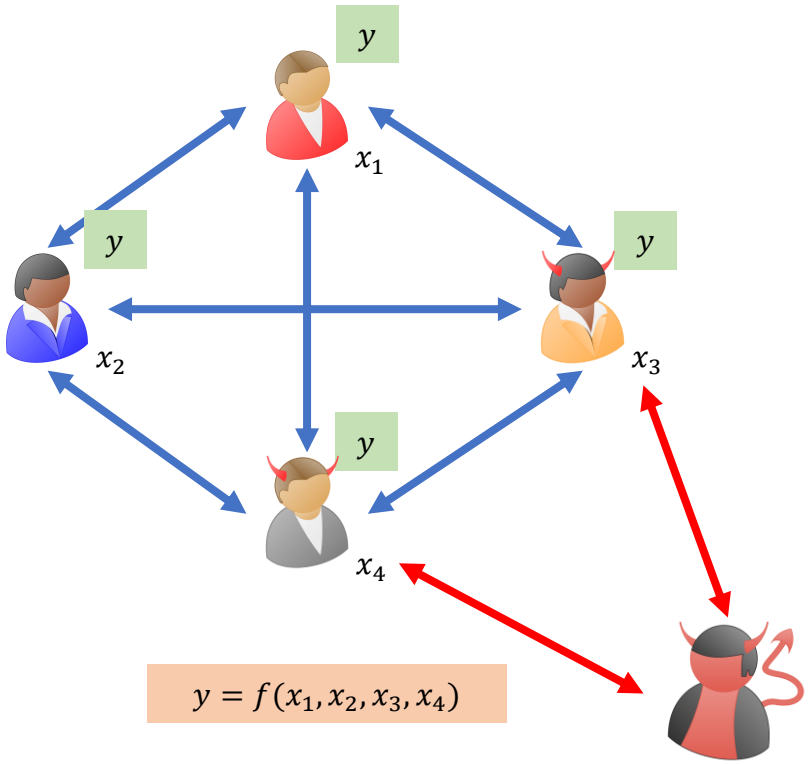
Covert Adversaries: Behaves maliciously if it can do so undetected.

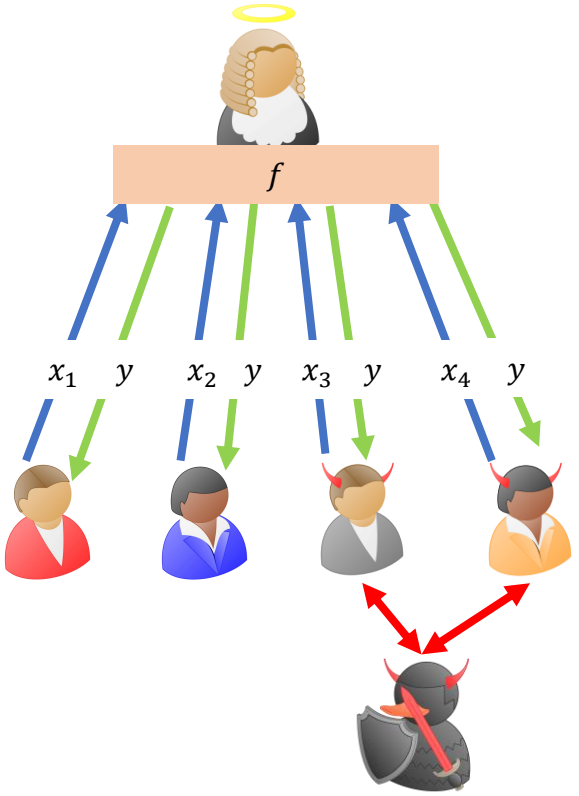
Covert Adversaries

[Aumann-Lindell'07]

real world



ideal world



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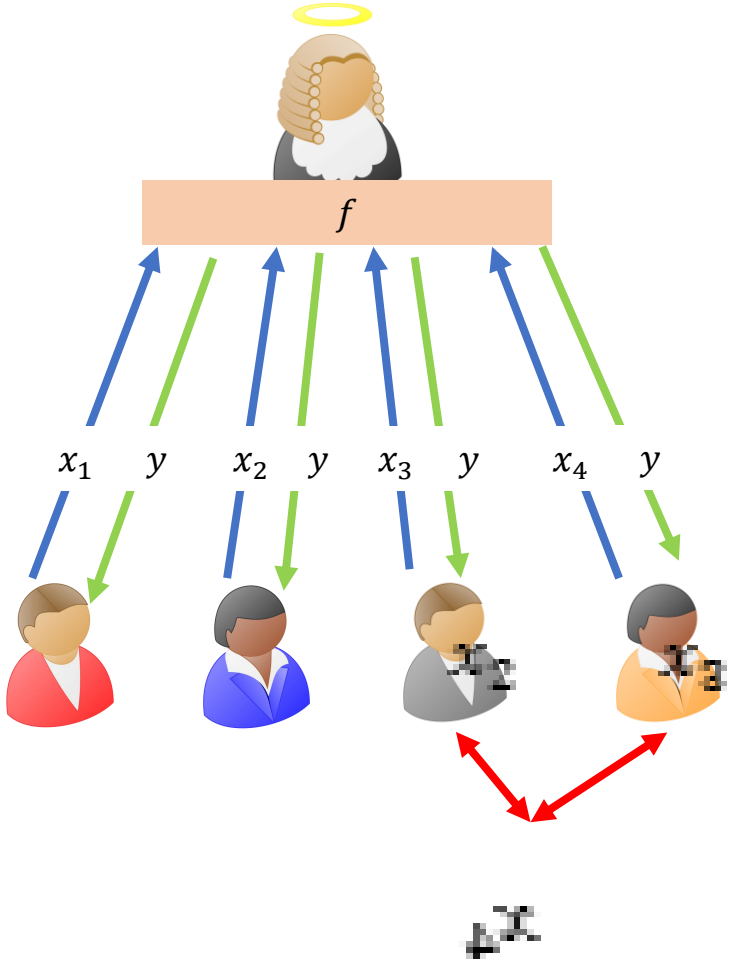
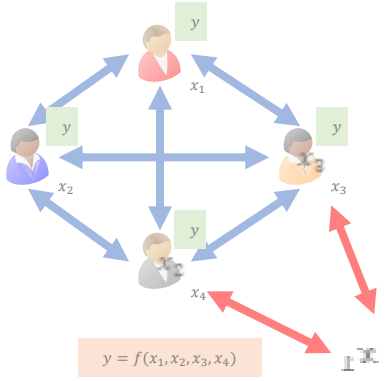


real world



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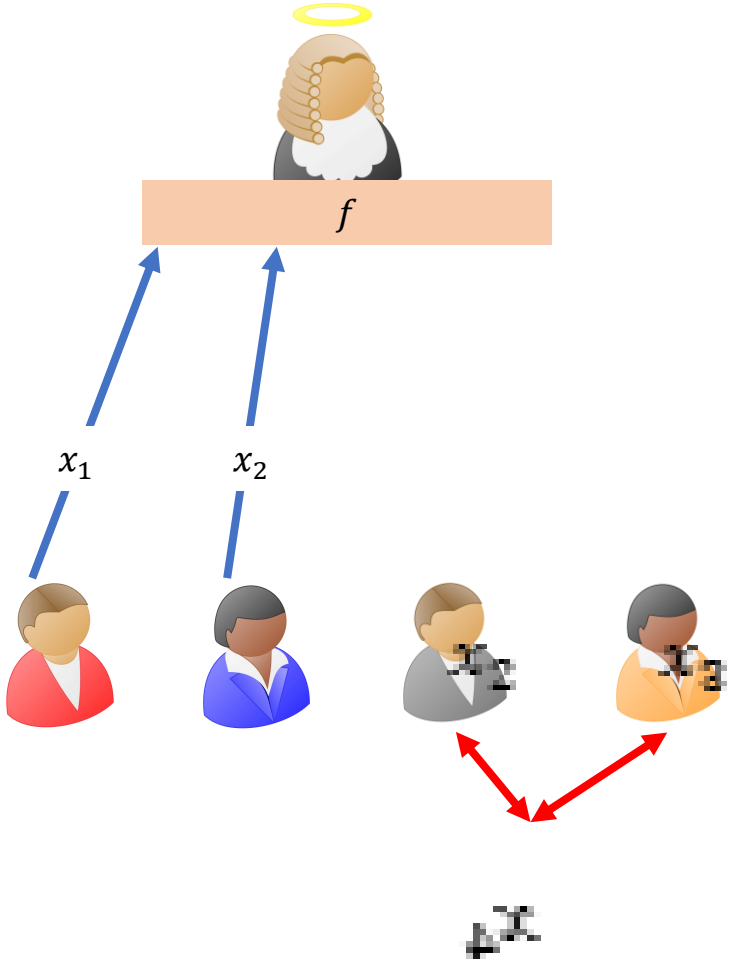
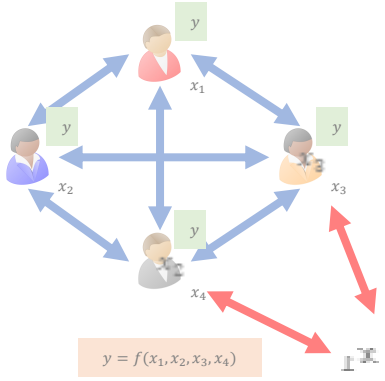


real world



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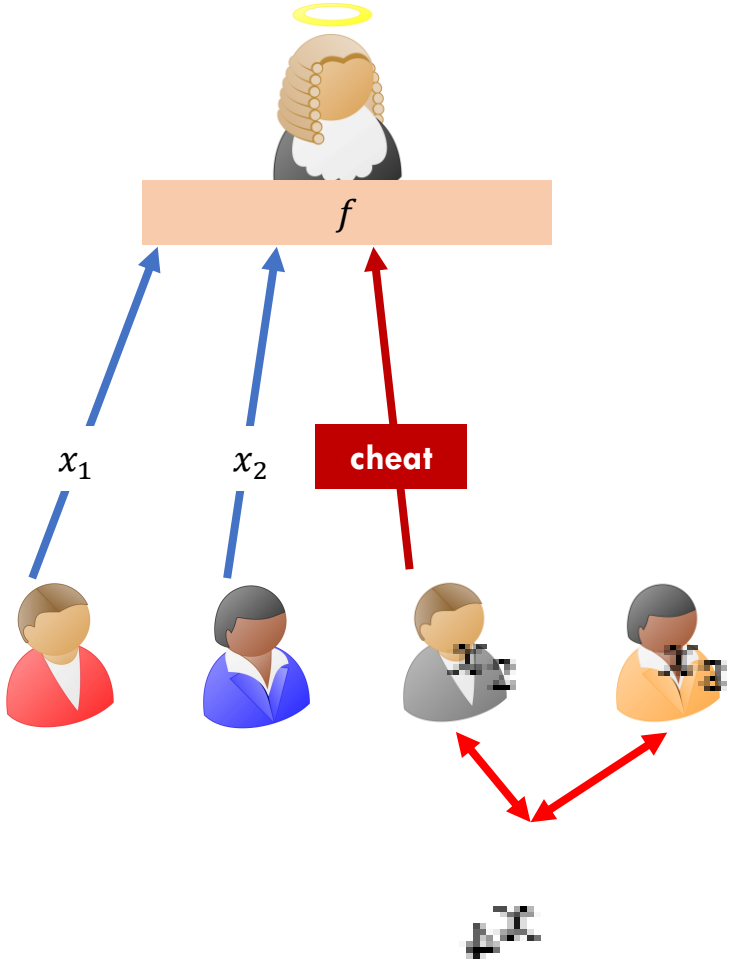

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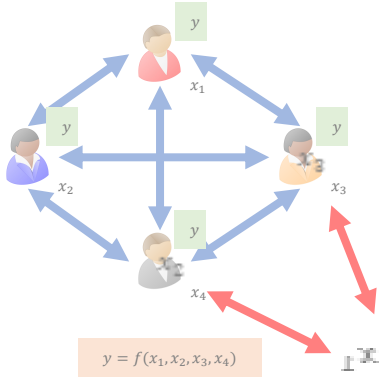

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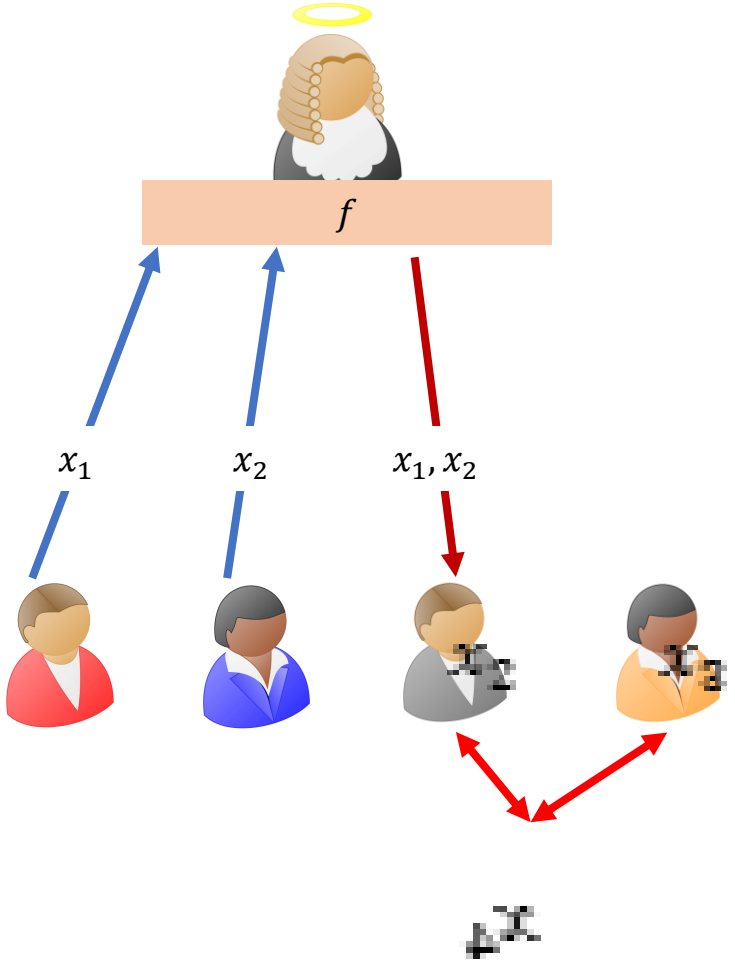

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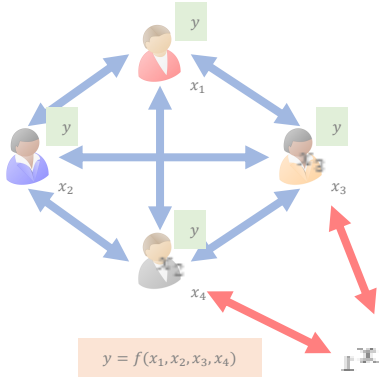

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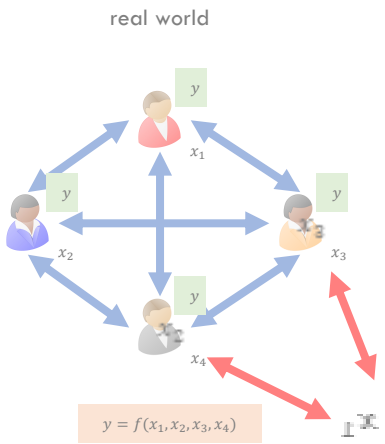

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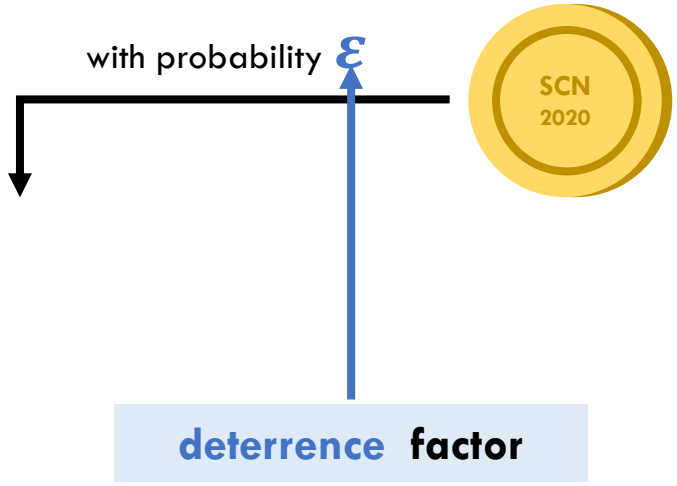

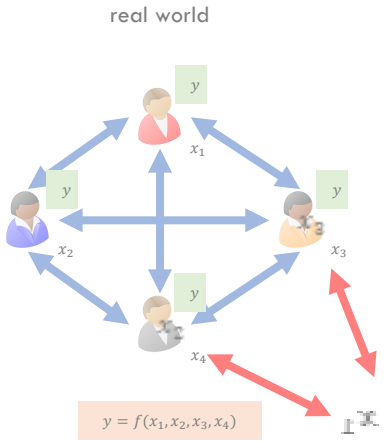

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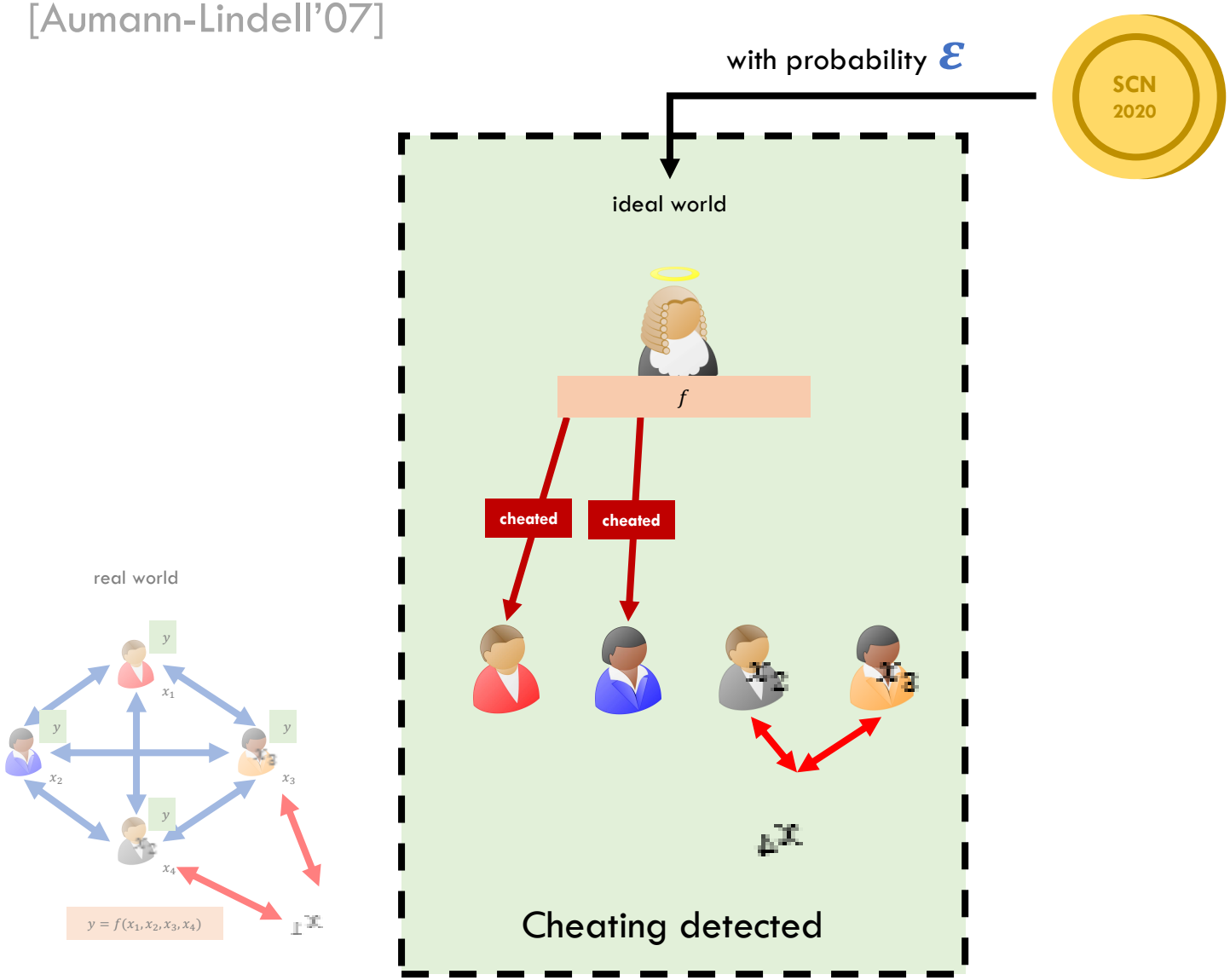

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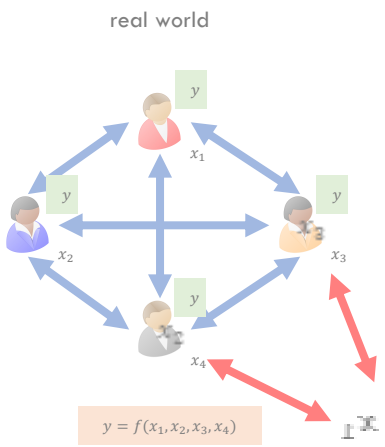
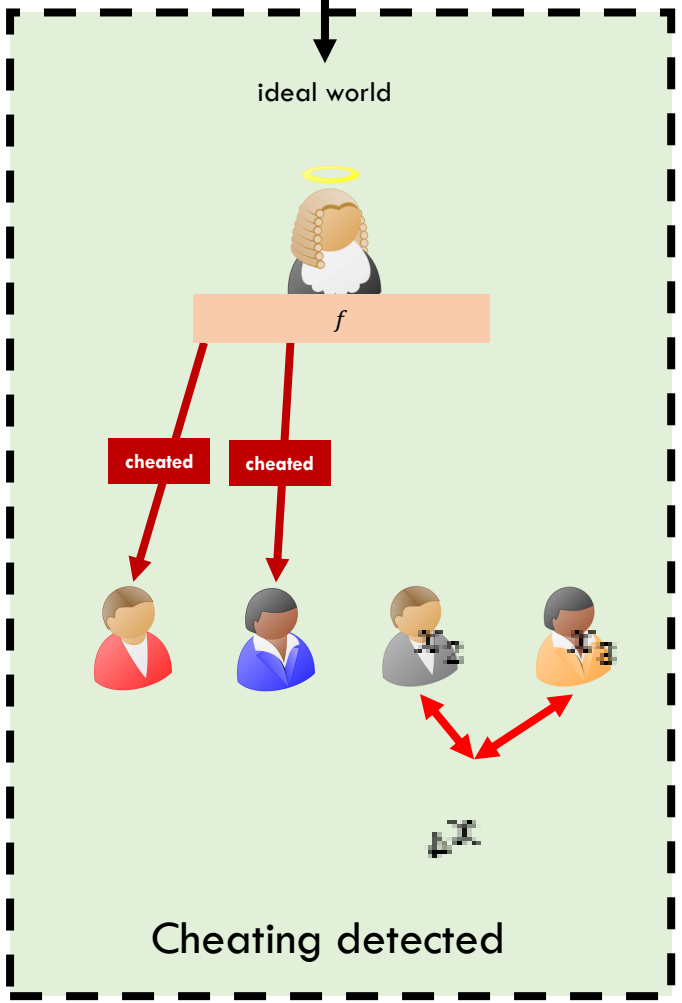
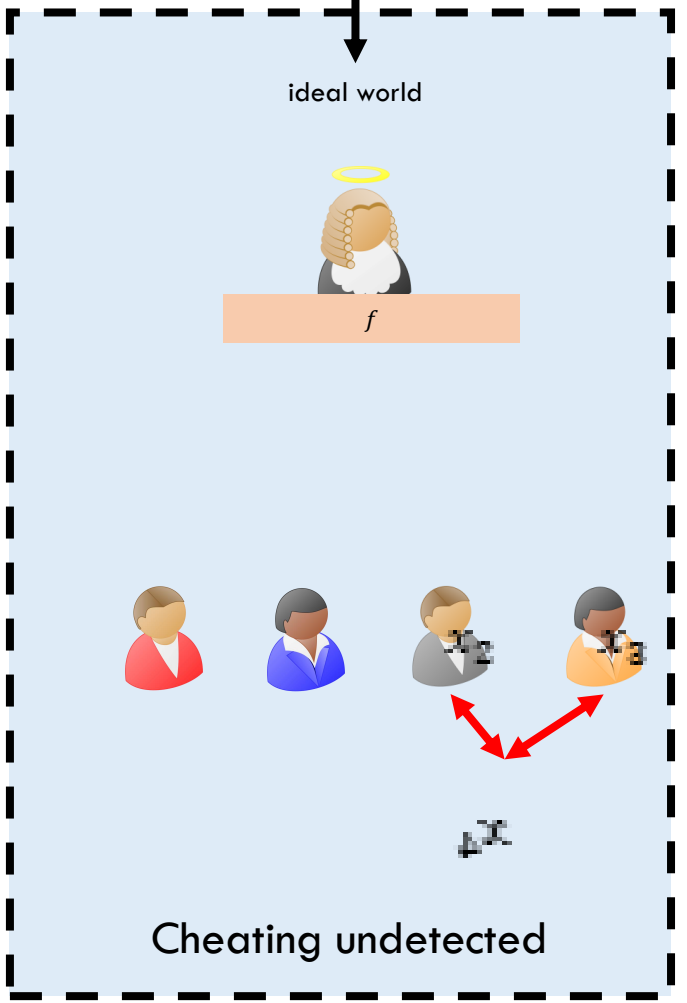


with probability ε

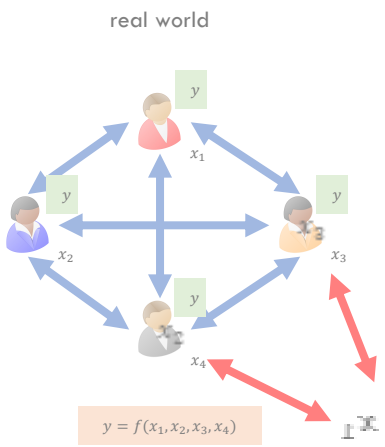
with probability $1 - \varepsilon$





Covert Adversaries

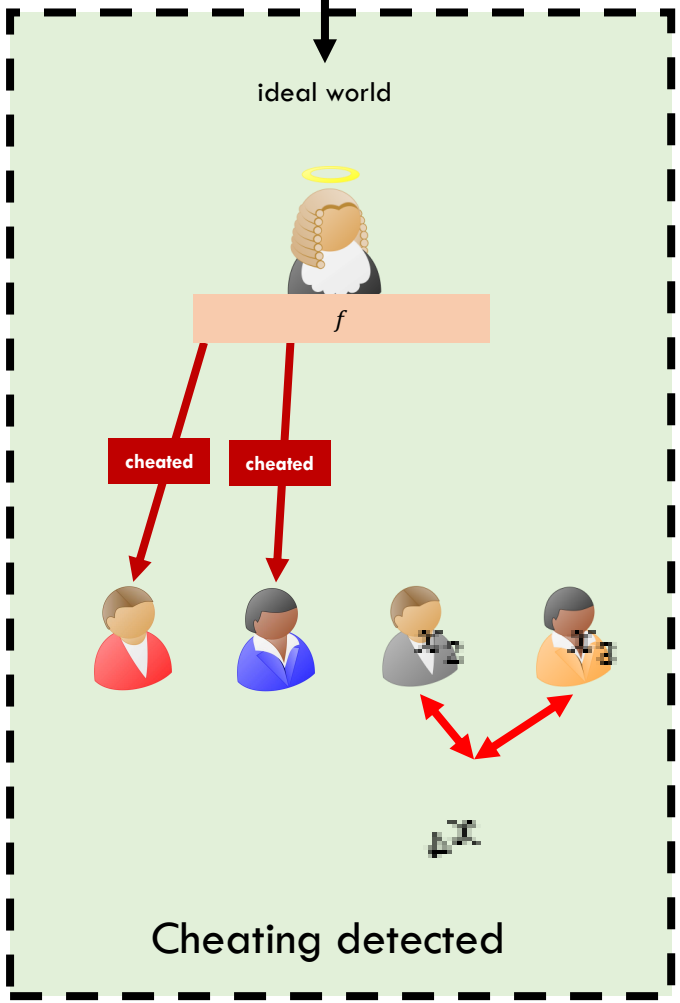
[Aumann-Lindell'07]

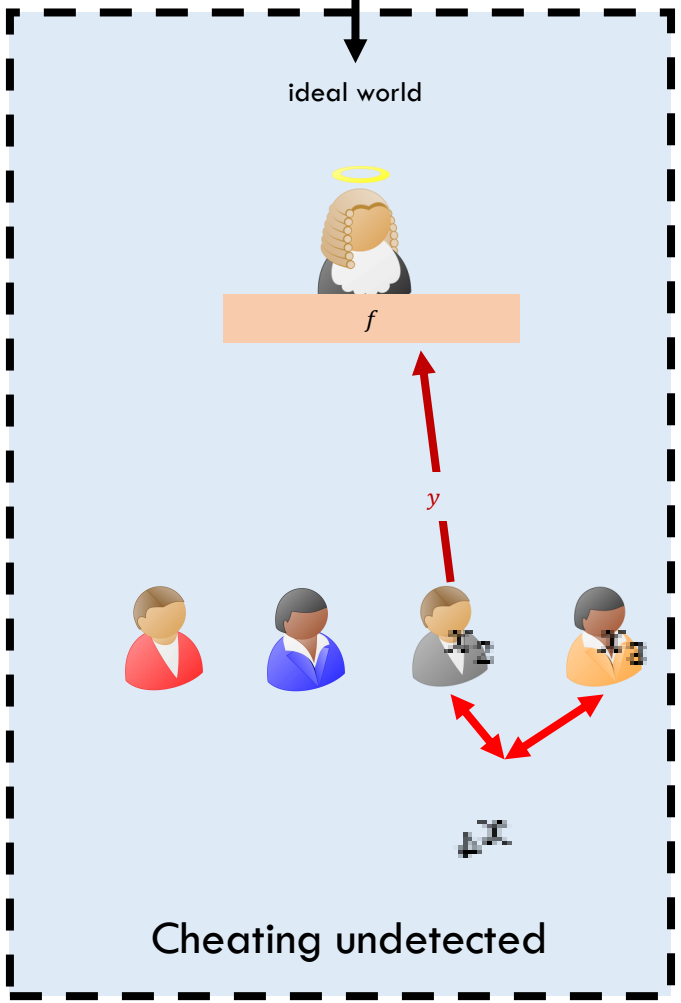


with probability ε

SCN 2020

with probability $1 - \varepsilon$





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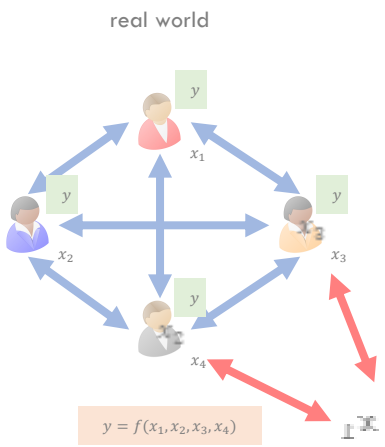

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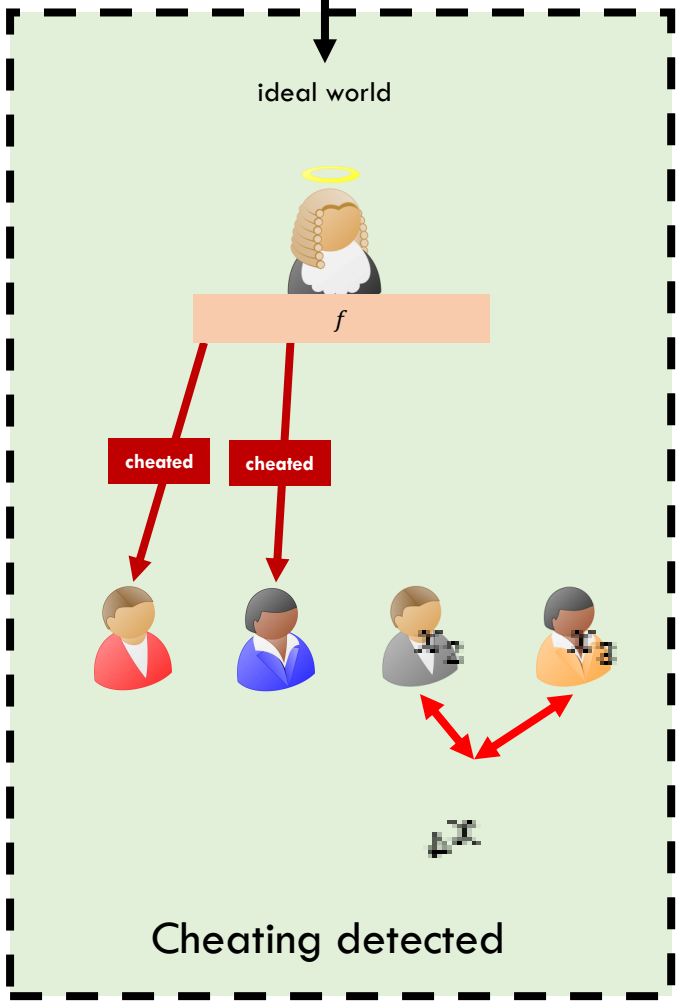

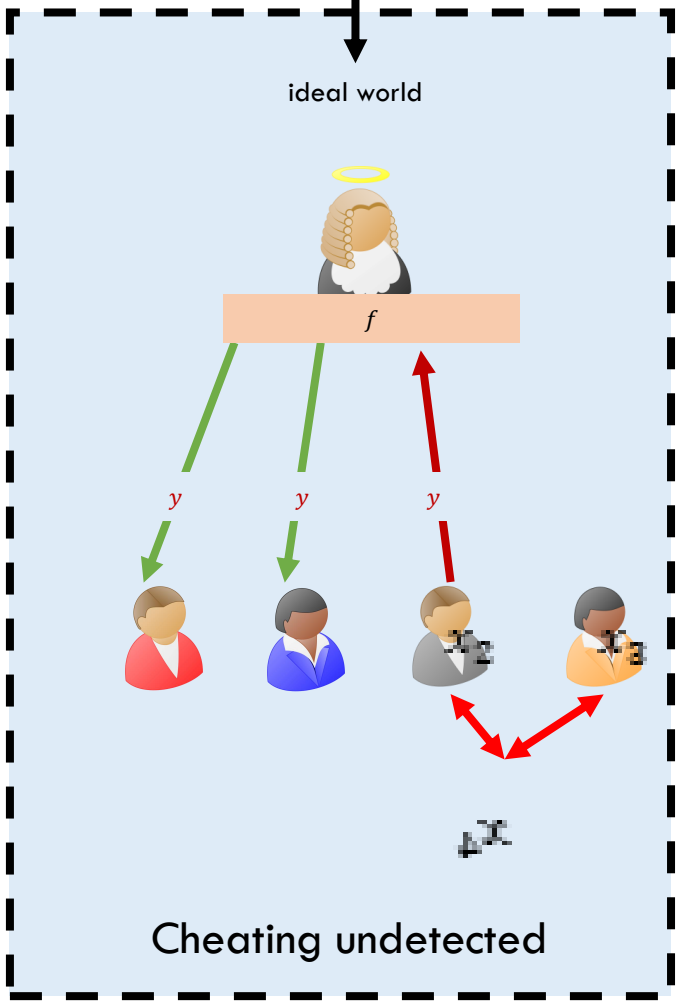

What is the precise round complexity of MPC protocols in the presence of covert adversaries?

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Best known results are those achieving malicious security, i.e. 4 rounds.

Prior works focus on computational complexity.

Our Results

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There does not exist a three round protocol in the presence of covert adversaries with respect to black-box simulation.

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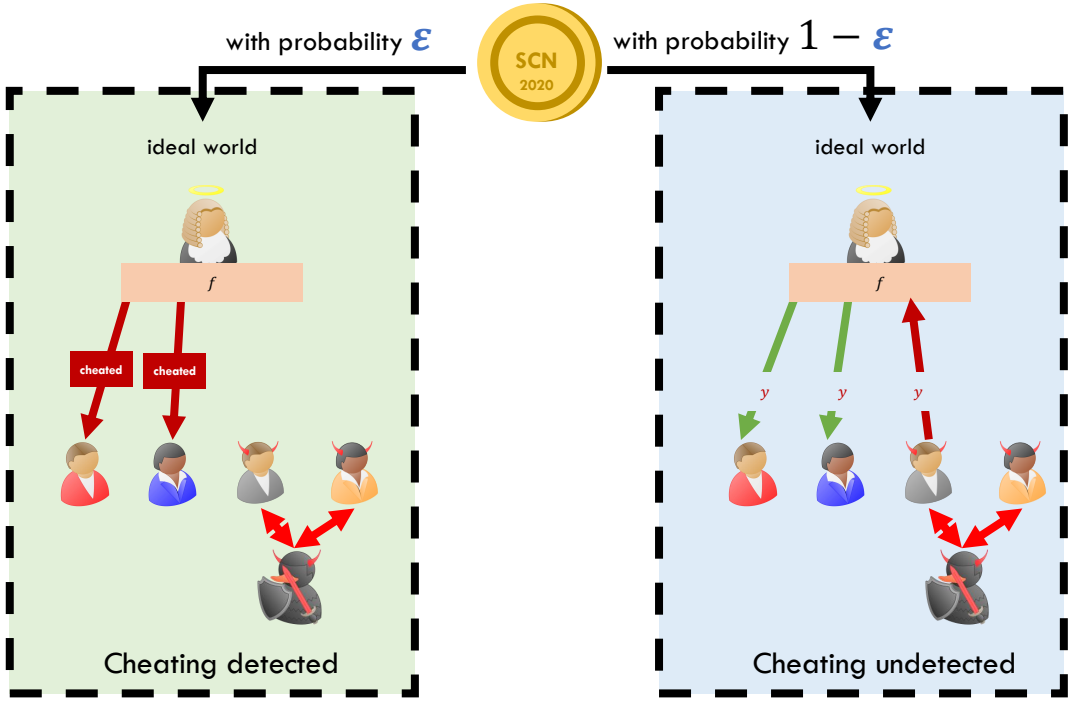
n – number of 

ε – deterrence factor 



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Best case:

2 rounds

$\varepsilon = 0$

semi-honest

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Worst case:
5 rounds
$\varepsilon \geq 1/2$

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There exists a variable round protocol in the presence of covert adversaries where the expected number of rounds are:

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Lower Bound

There does not exist a three round protocol in the presence of covert adversaries with respect to black-box simulation.

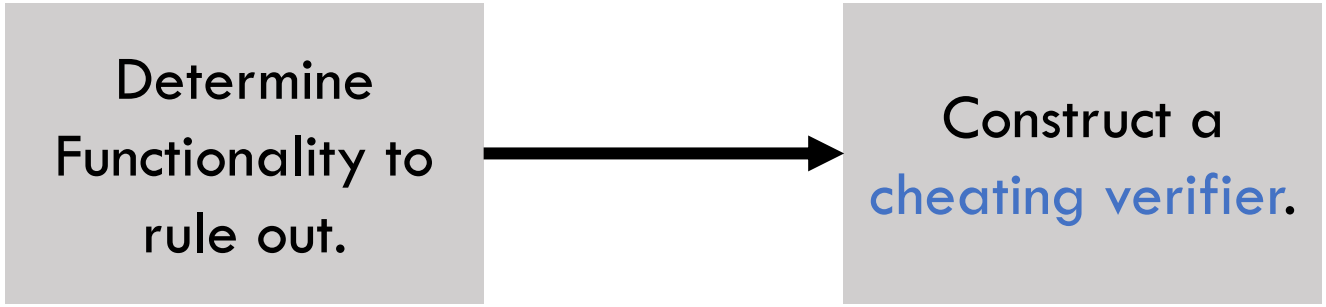
High Level Strategy

High Level Strategy

Determine Functionality to rule out.

Two Party **Zero-knowledge(ZK)** functionality

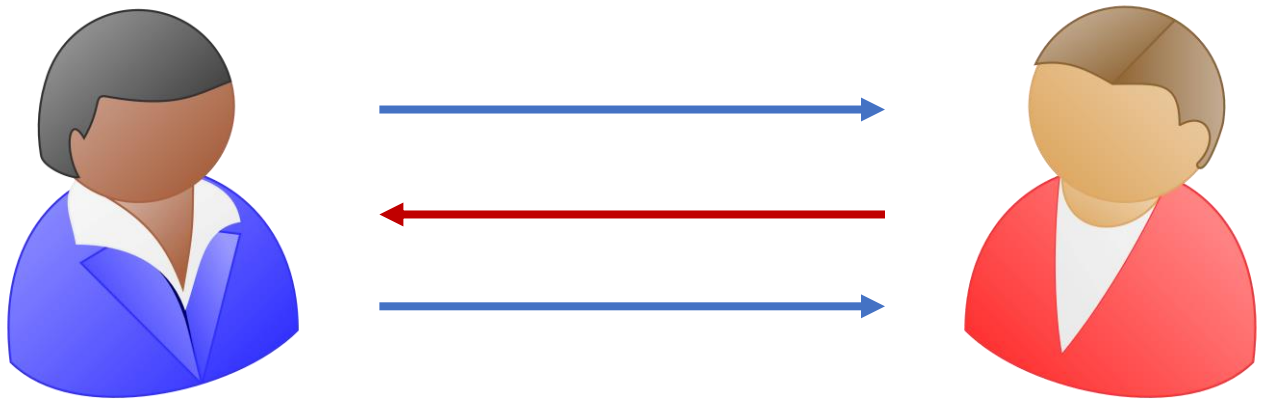
High Level Strategy



Two Party **Zero-knowledge(ZK)** functionality

High Level Strategy

Two Party 3 round Zero-Knowledge protocol

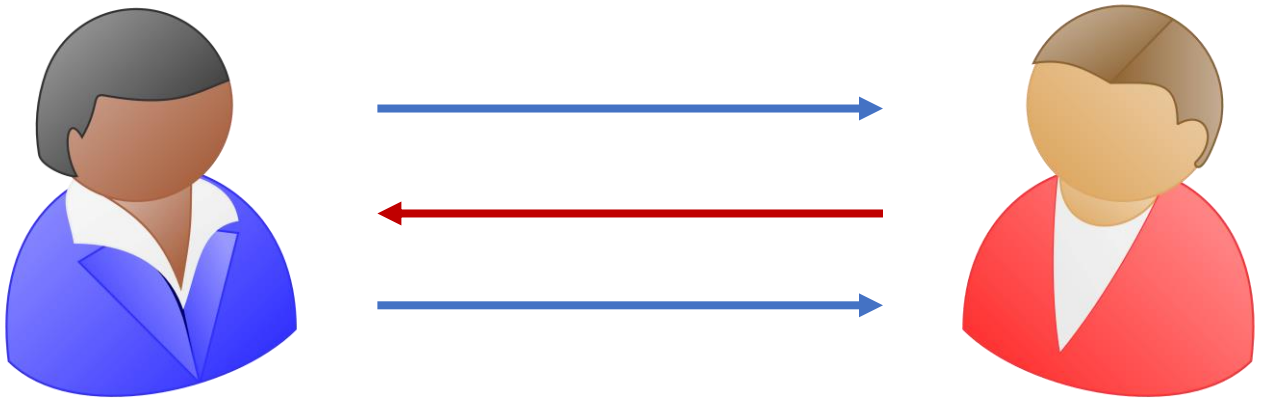


prover
(x, w)

verifier
(x)

Two Party 3 round Zero-Knowledge protocol

[Goldreich-Krawczyk'96]
3 round ZK
does not exist



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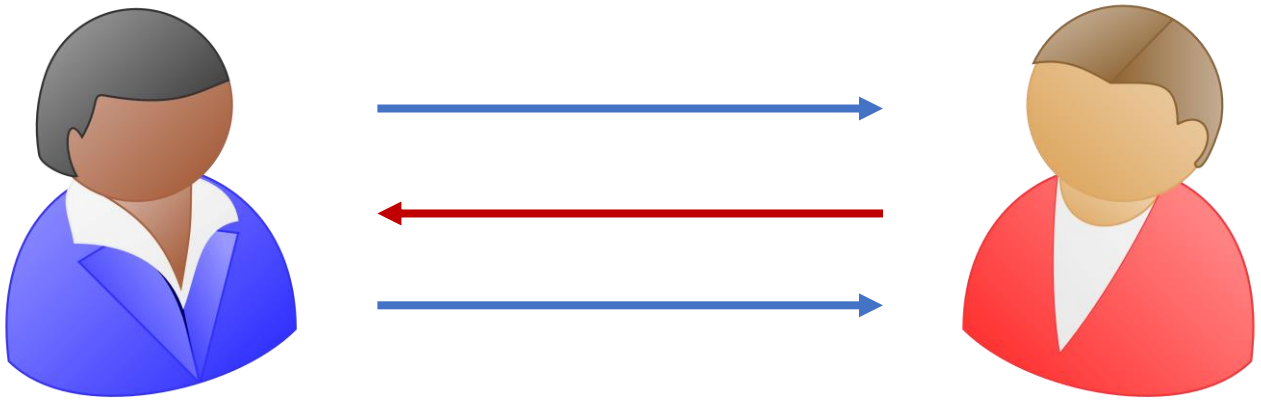

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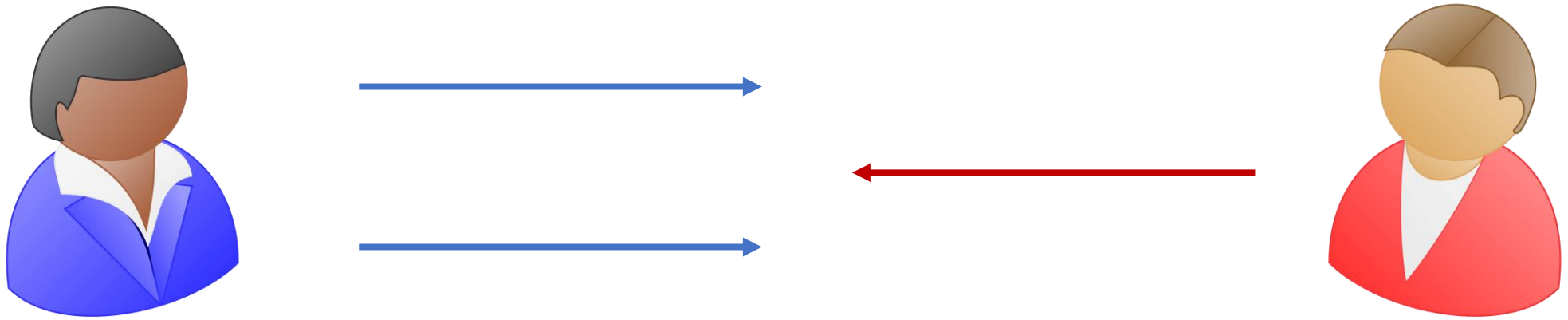
Why doesn't [Goldreich-Krawczyk'96] directly apply?

Challenge 1: Simultaneous Messages



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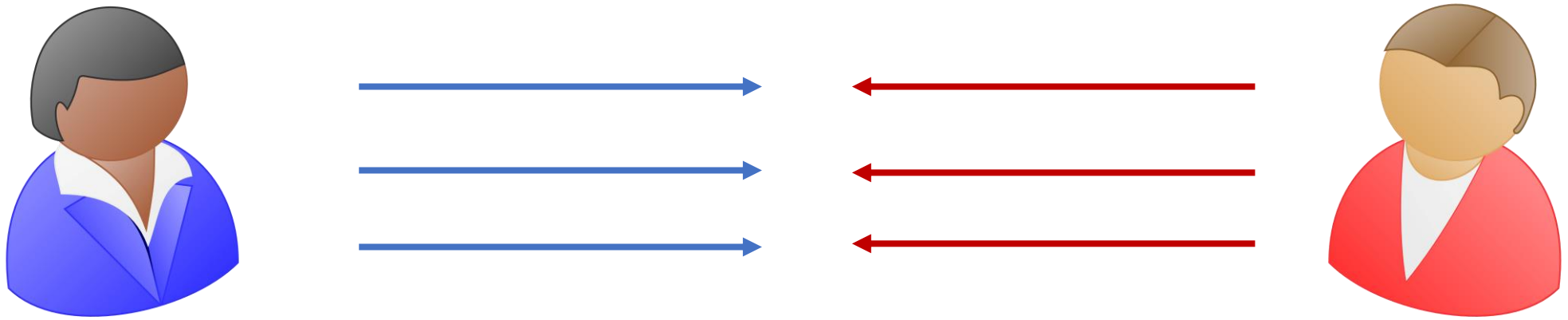

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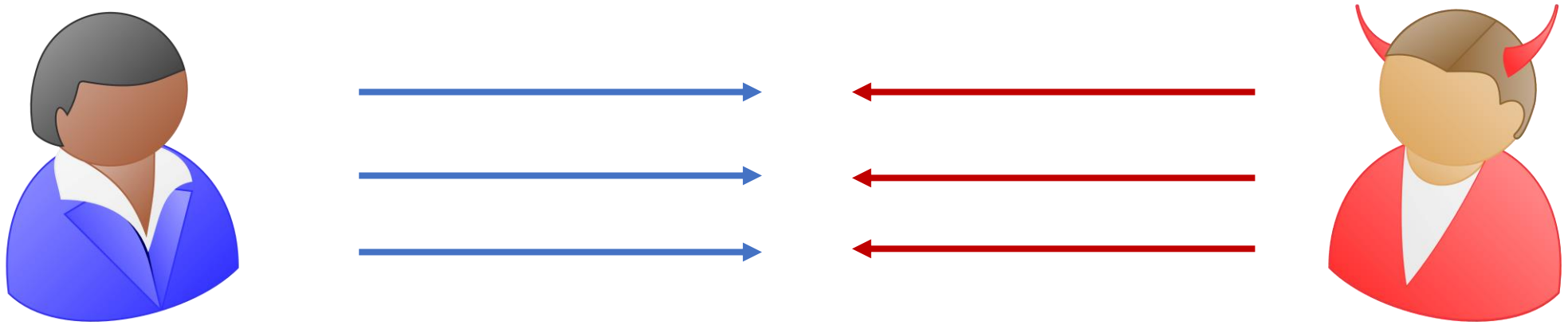
Simulator can potentially use all 3 messages sent by Bob to extract a trapdoor, and complete simulation.

Challenge 2: Covert Adversary



prover
(x, w)

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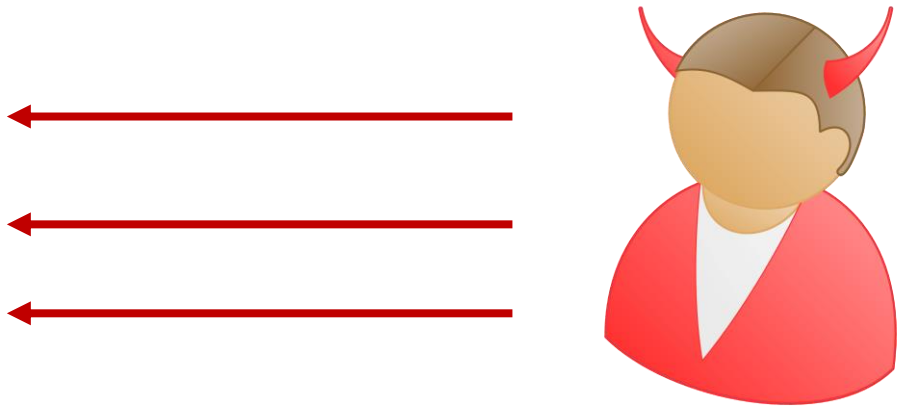
If Simulator determines Bob is cheating, can query with cheat message to get Alice's input, witness w.

The (Rushing) Cheating Verifier



verifier
(x)

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Cheating Bob can choose its i-th round message after receiving the i-th round message from the prover.

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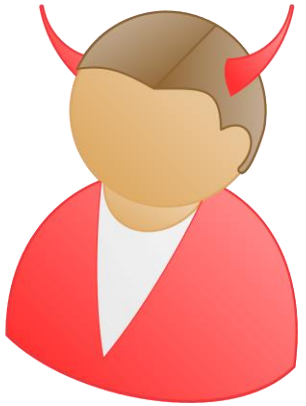

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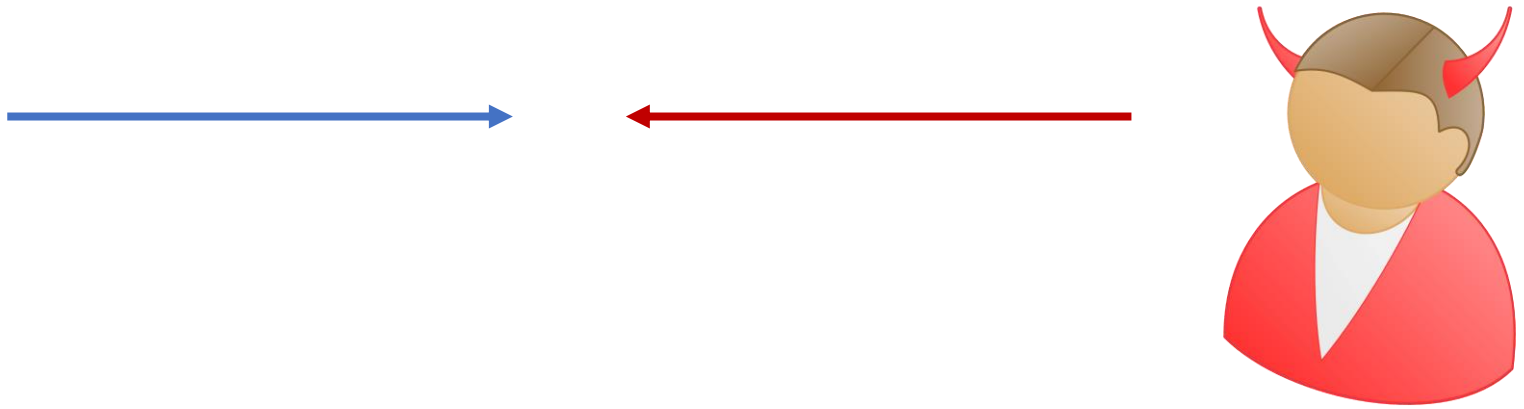

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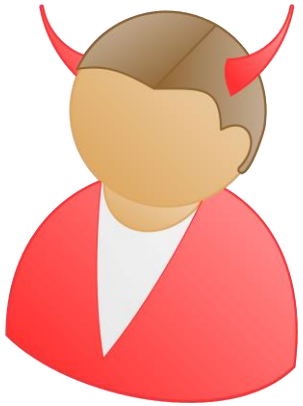
Intuition:

Behave honestly in the first two rounds using fresh randomness for each run.



verifier
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Intuition:

1) Using fresh randomness prevents Simulator from rewinding first two rounds.

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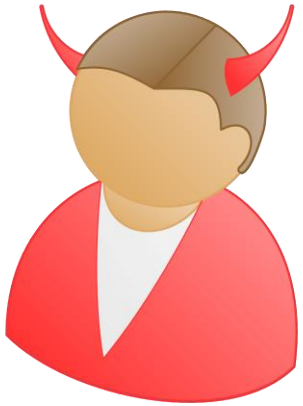

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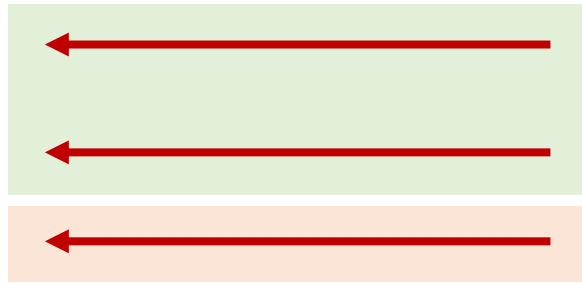
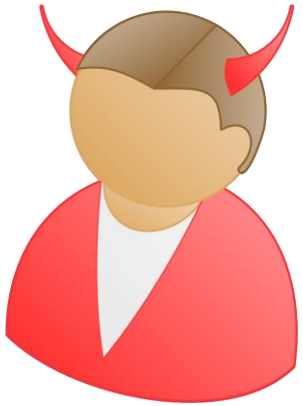


Send third round message if received third round is accepting, else abort.

Rushing Covert Verifier



verifier (x)

The (Rushing) Cheating Verifier

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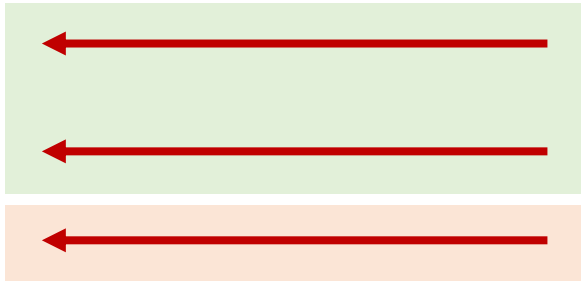

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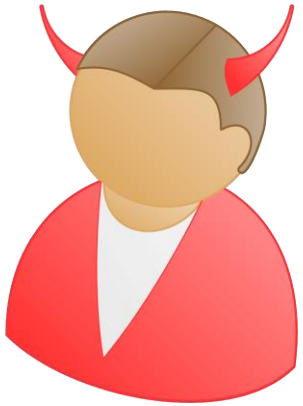


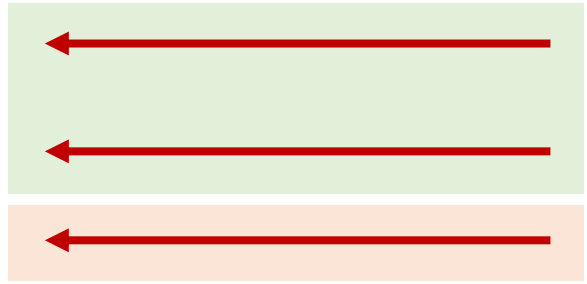
verifier (x)

The (Rushing) Cheating Verifier

Intuition:

1) Using fresh randomness prevents Simulator from rewinding first two rounds.
2) Third round strategy prevents simulator from receiving Bob's third round unless it has an accepting transcript.
3) Bob will not cheat in an honest execution with the real prover $\Longrightarrow$ simulator cannot send cheat query.

Behave honestly in the first two rounds using fresh randomness for each run.



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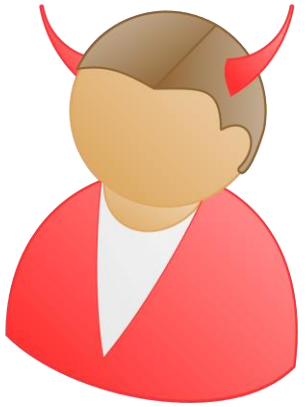

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Upper Bound

There exists a variable round protocol in the presence of covert adversaries where the expected number of rounds are:

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Upper Bound

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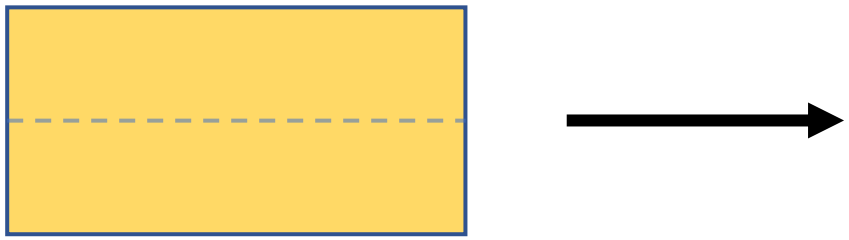
Covert Adversary Compiler



Two round protocol secure
against adversarial randomness

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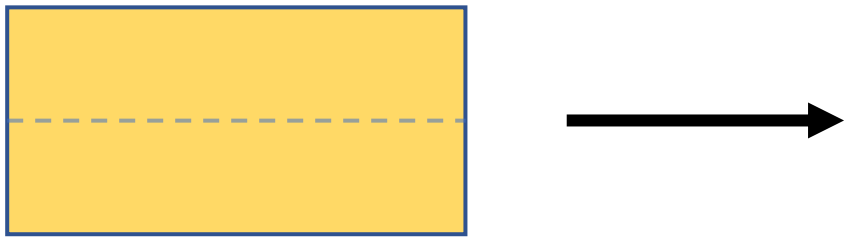

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Main Idea: Initiate check for honest behavior with certain probability.

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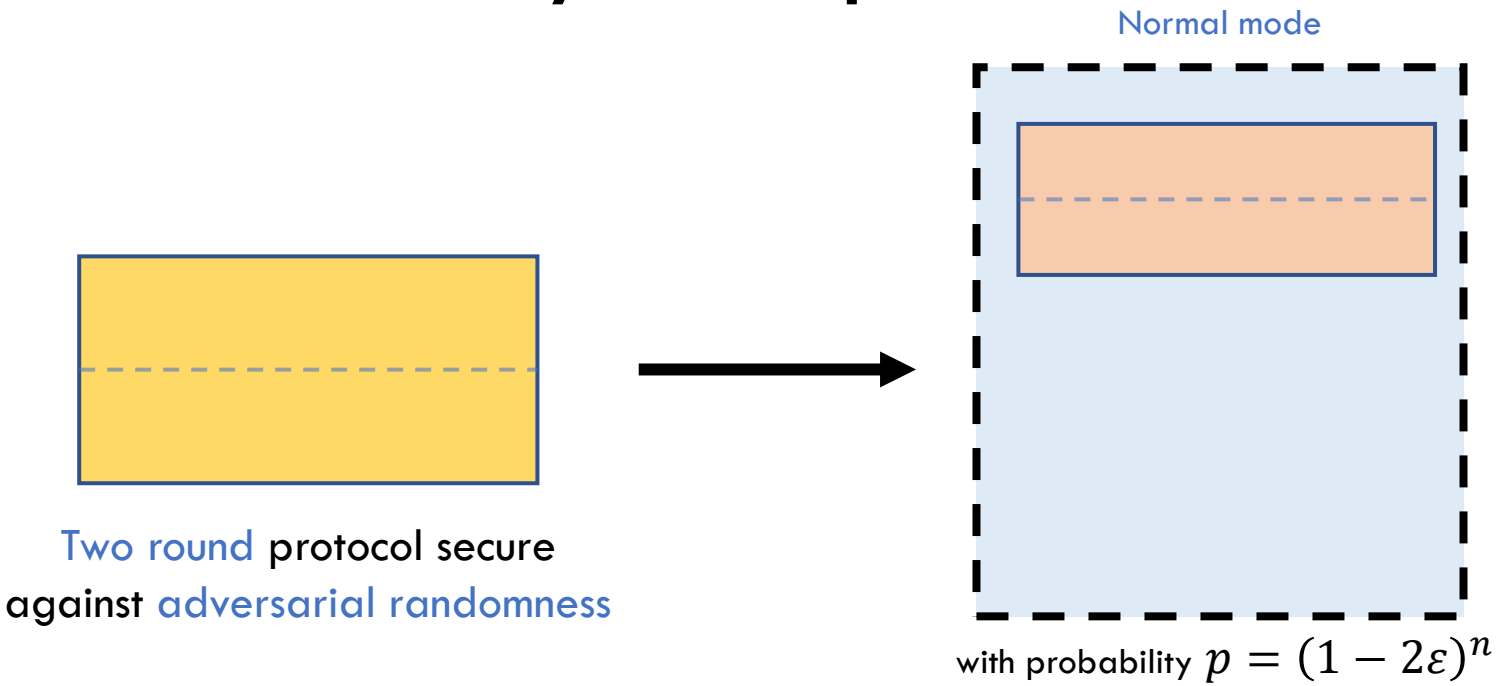

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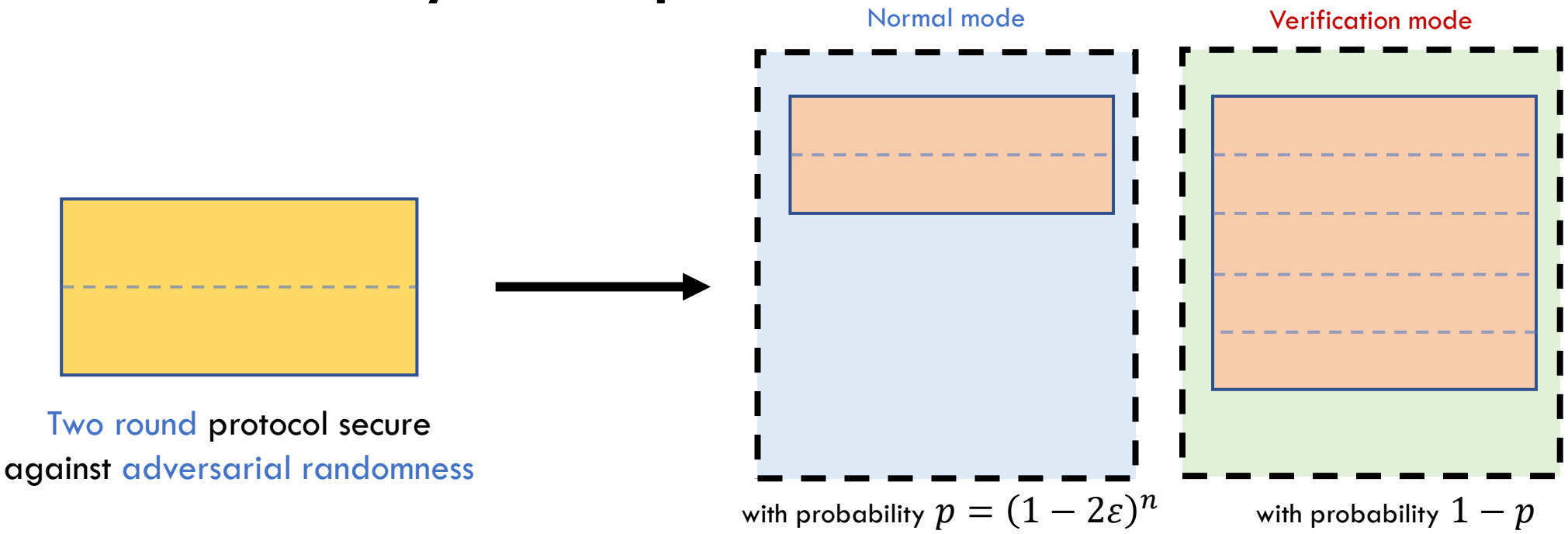

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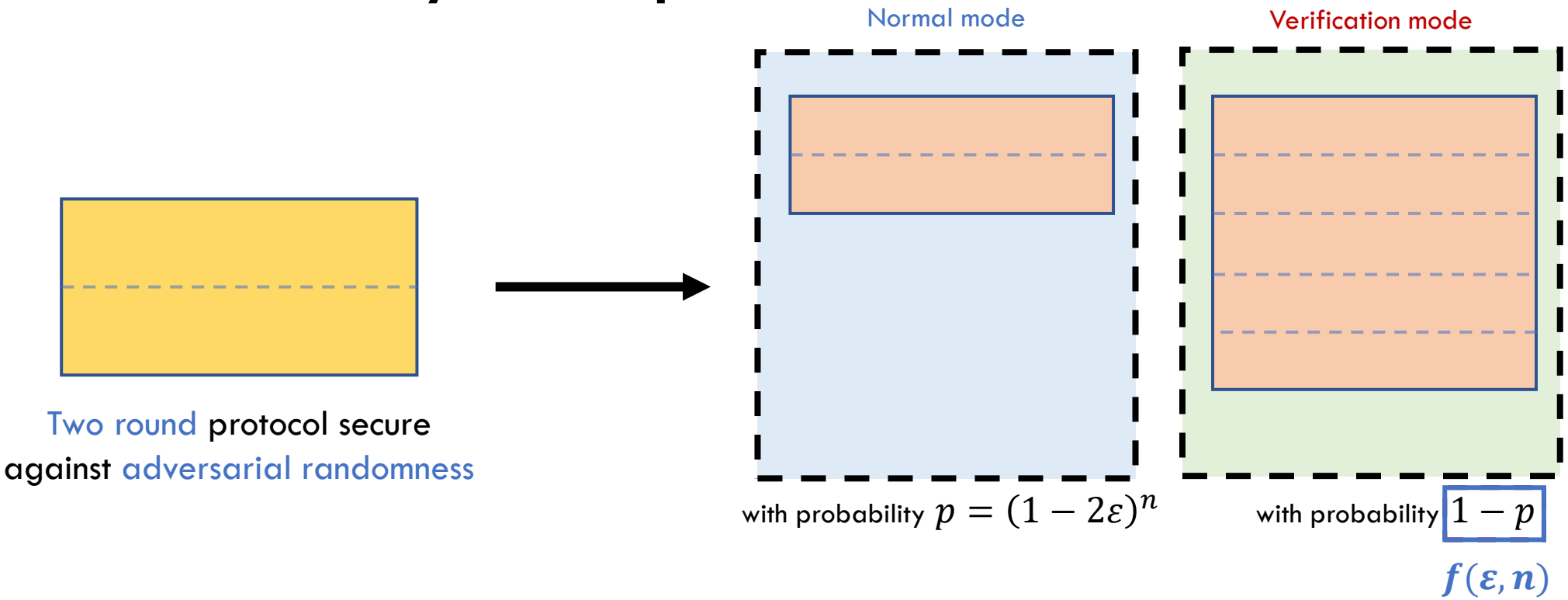

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High Level Idea of the Protocol

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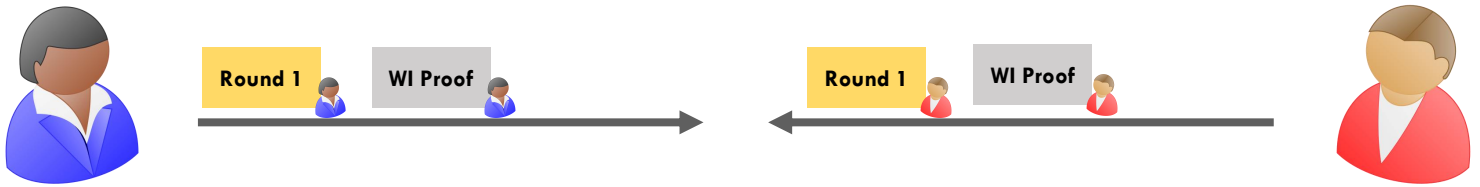

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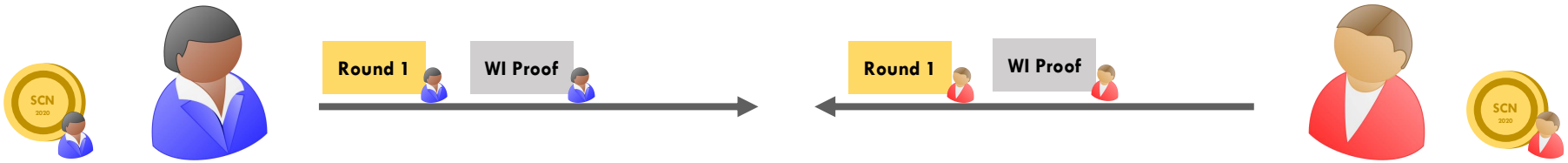

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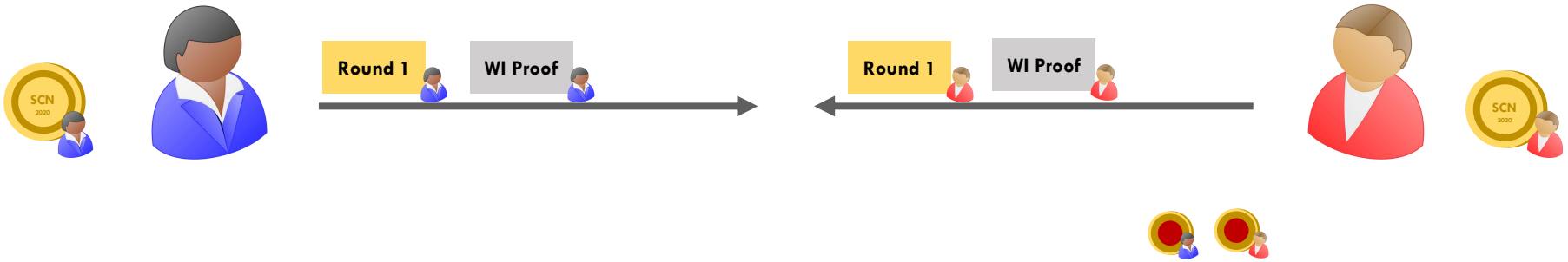

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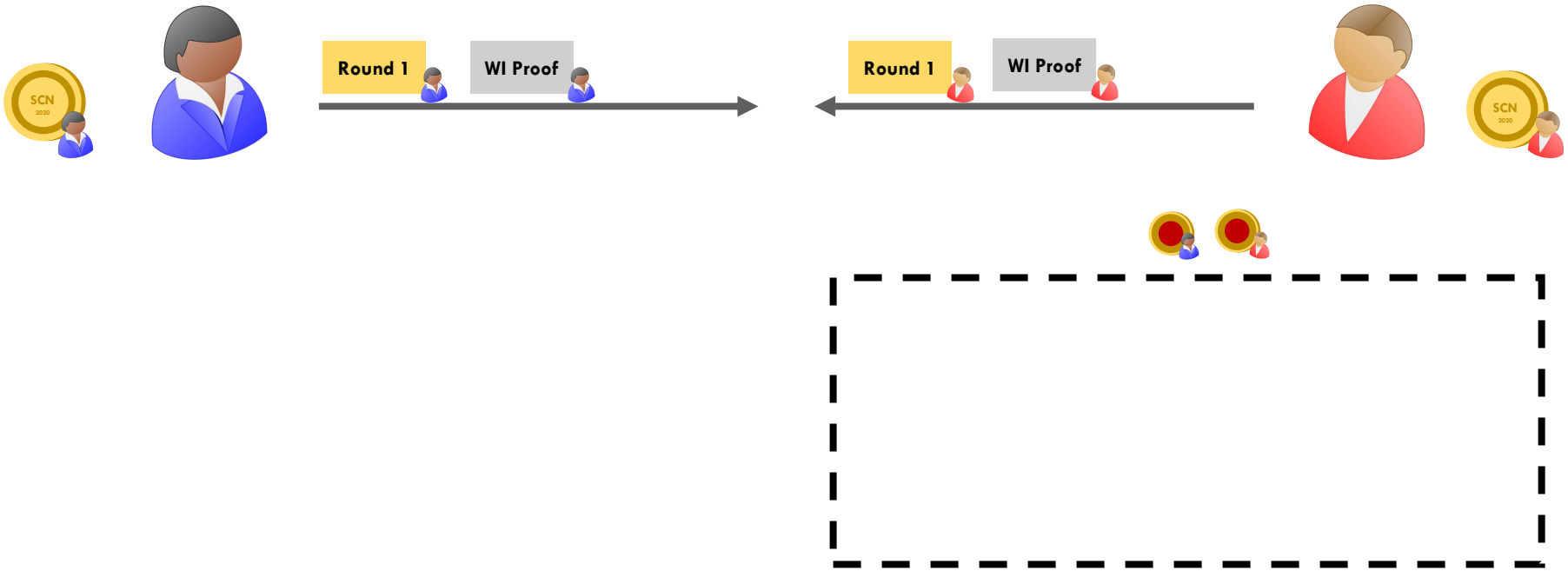

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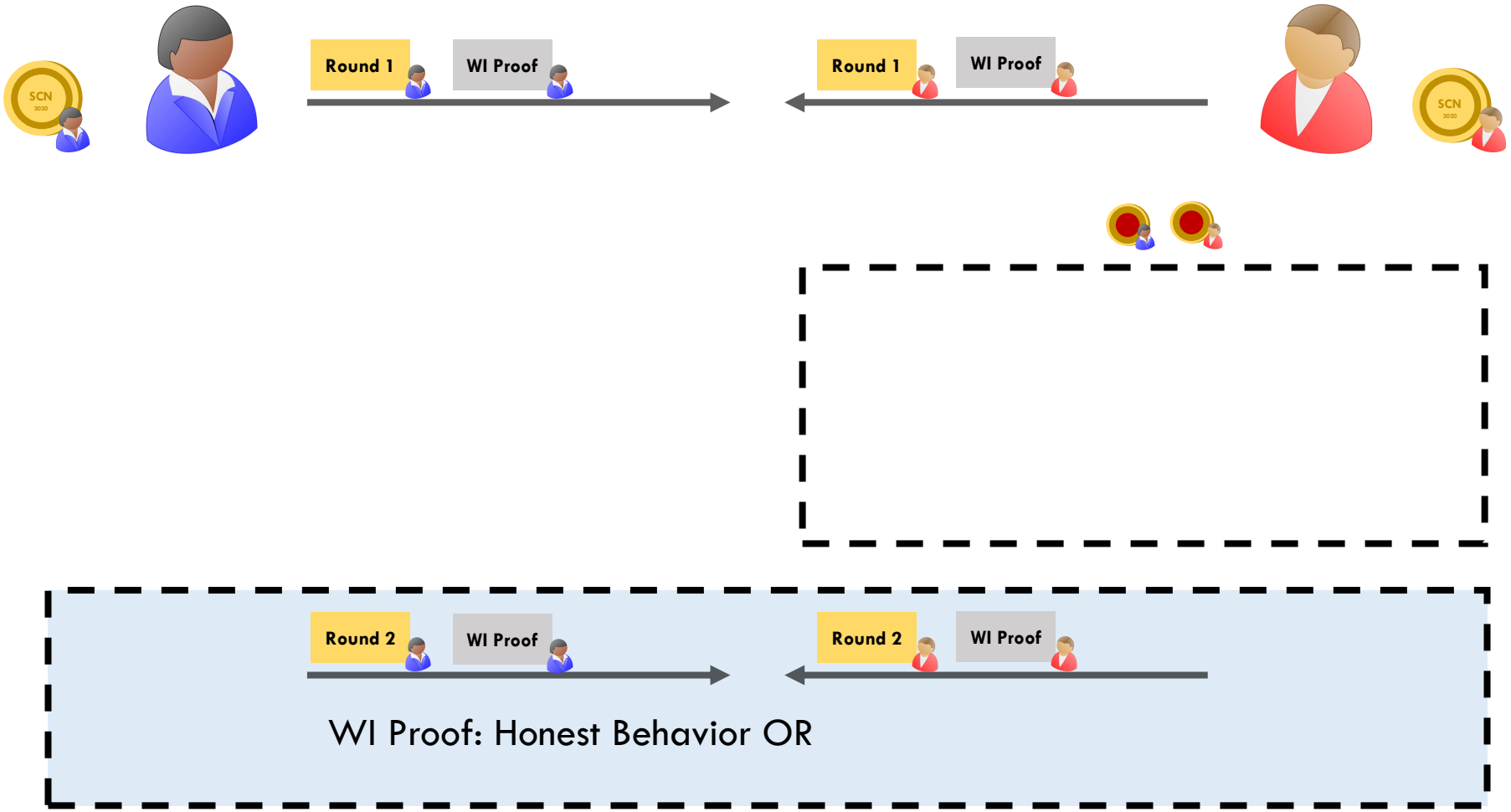

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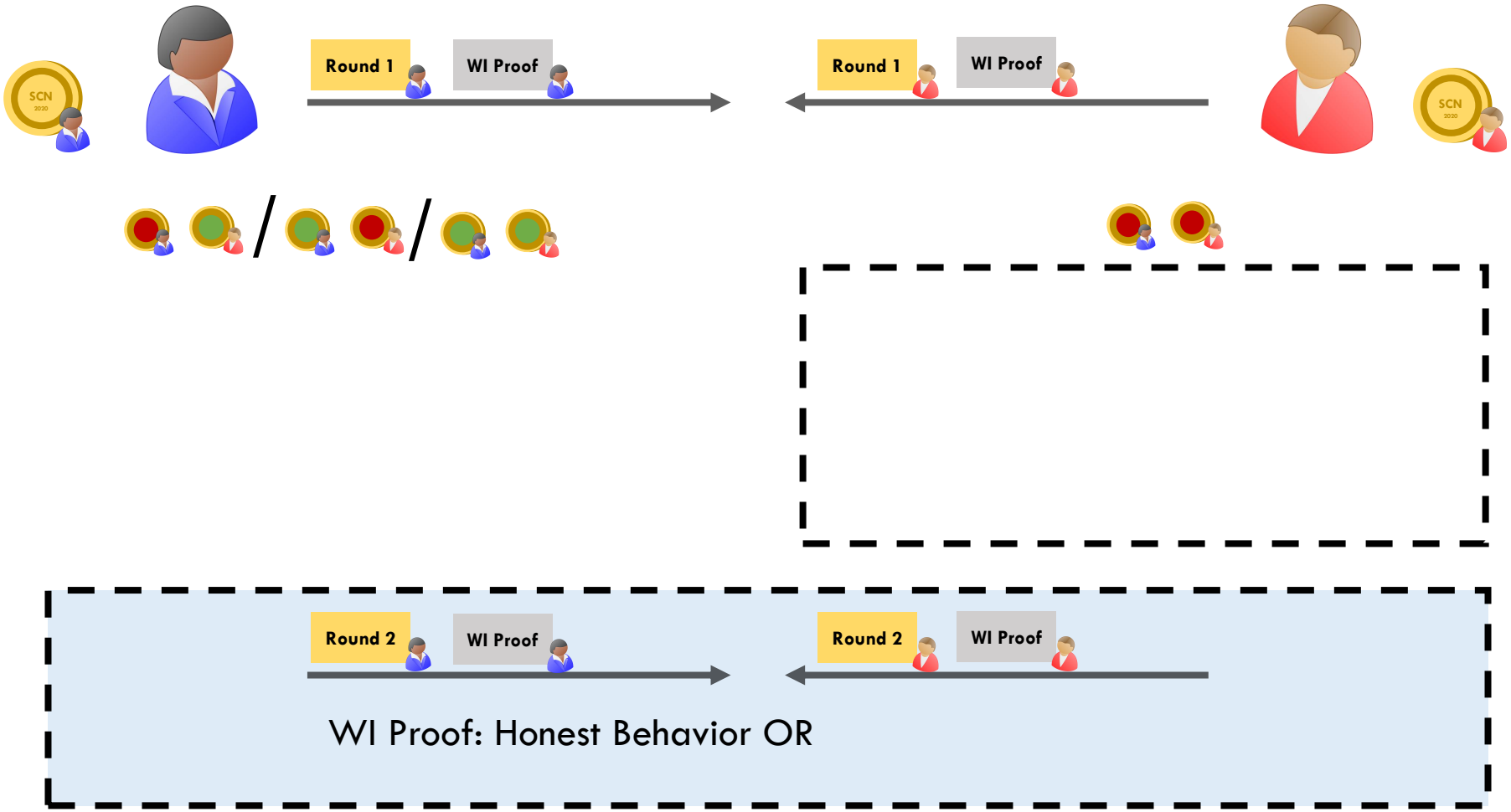

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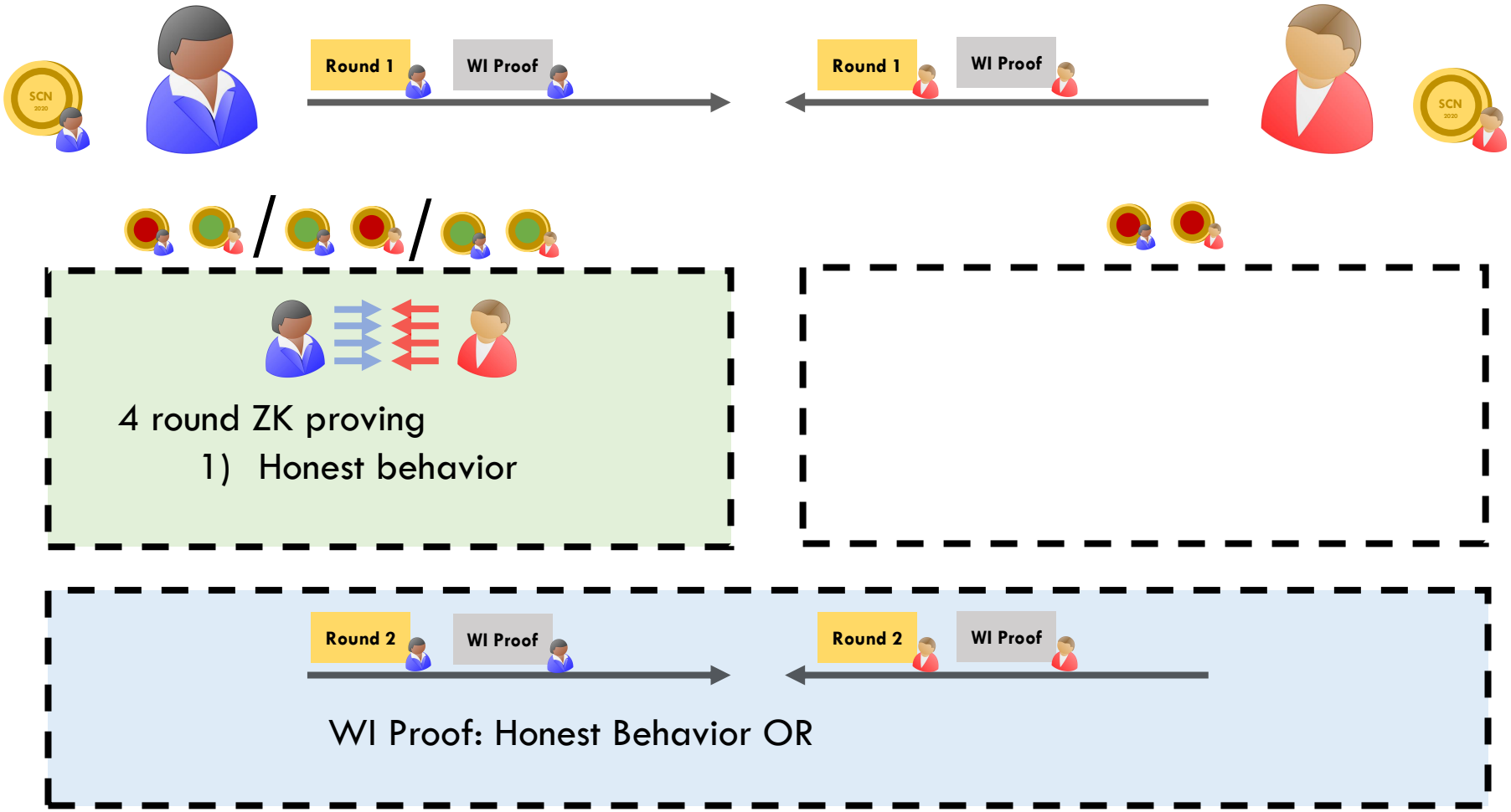

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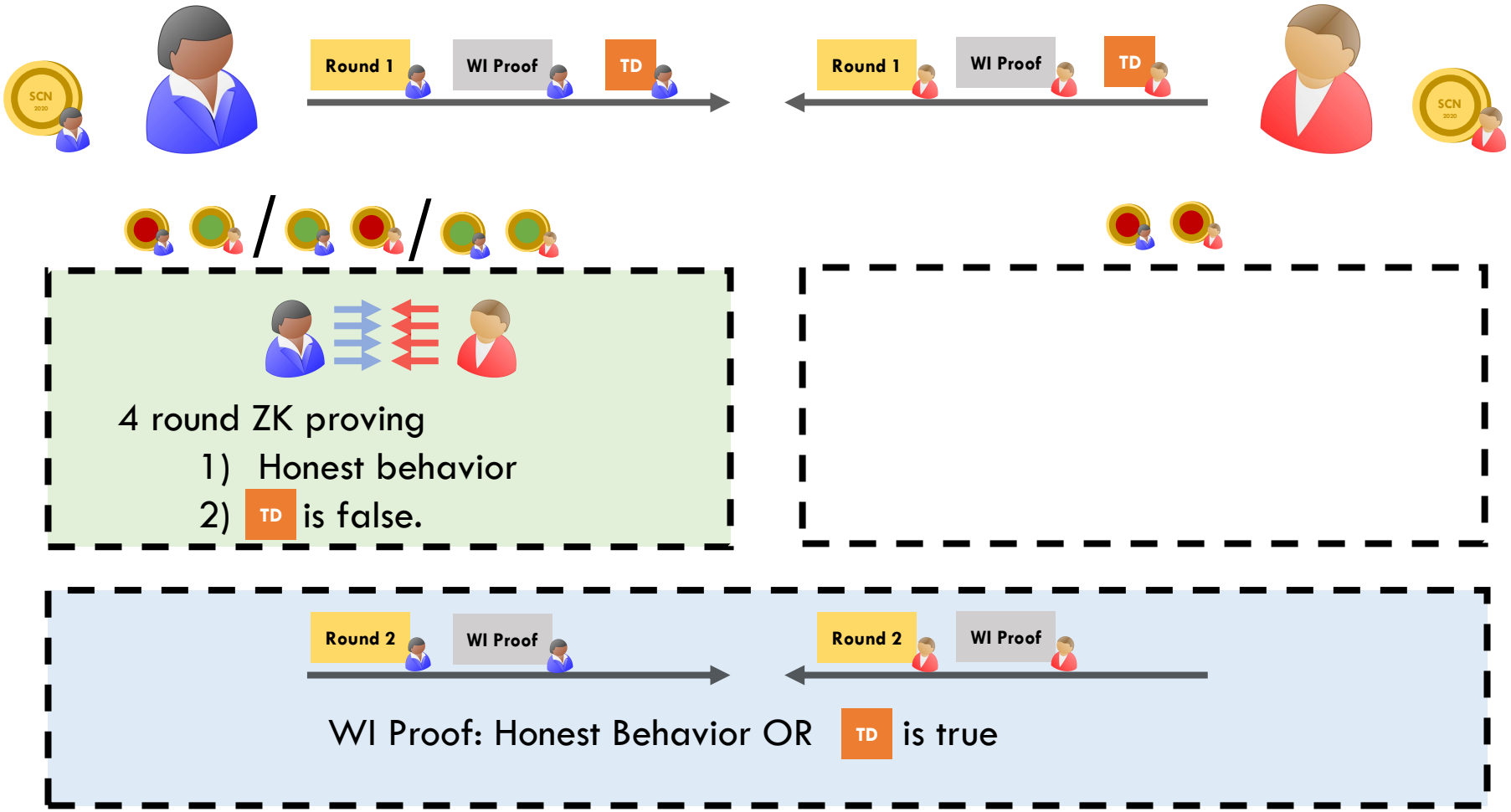
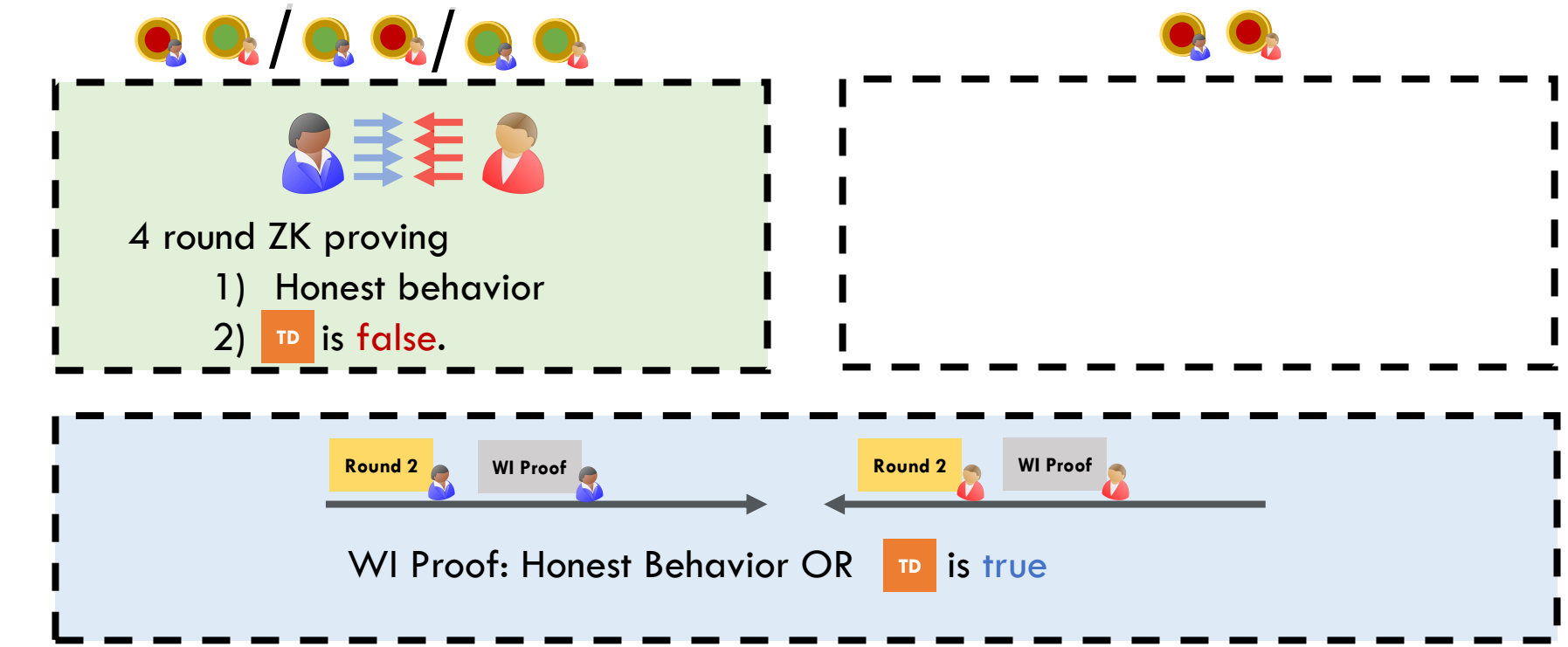


Intuition: Adversary must make a choice to set the trapdoor true to cheat in normal mode or risk being caught for incorrectly setting trapdoor in verification mode.



Several Simulation Challenges

For two rounds in best case:

Need stronger security from underlying protocol: free-simulation

Non-malleability issues

Skewed distribution of transcripts while checking if adversary is cheating

Worst case:

5 rounds

$\varepsilon \geq 1/2$

There exists a variable round protocol in the presence of covert adversaries where the expected number of rounds are:

$$2 + 3 \cdot (1 - (1 - 2\varepsilon)^n)$$

Worst case 4 rounds?

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- Current 4 round protocols complex

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Adversary forcing worst case?

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Similar ideas might likely work

Current 4 round protocols complex

Adversary forcing worst case?

Won't be able to cheat covertly

Worst case:

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There exists a variable round protocol in the presence of covert adversaries where the expected number of rounds are:

$$2 + 3 \cdot (1 - (1 - 2\varepsilon)^n)$$

There exists a protocol with expected number of rounds $2 + 3 \cdot (1 - (1 - 2\varepsilon)^n)$

There does not exist a three-round protocol.

There exists a protocol with expected number of rounds $2 + 3 \cdot (1 - (1 - 2\varepsilon)^n)$

There does not exist a three-round protocol.

Thank you. Questions?

Arka Rai Choudhuri

achoud@cs.jhu.edu