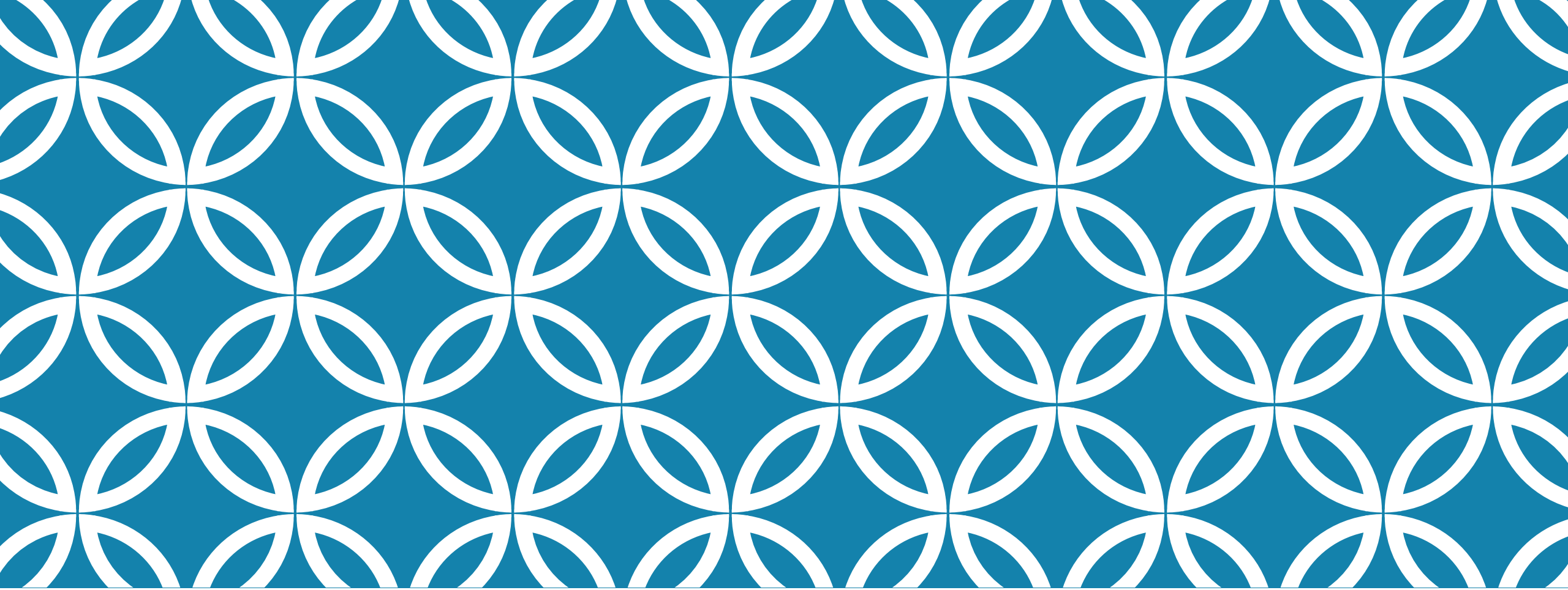

KEY-AND-ARGUMENT-UPDATABLE QA-NIZKS

Helger Lipmaa
Simula UiB, Bergen, Norway

REMINDER: NIZK

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$\pi = P(x, w)$

π

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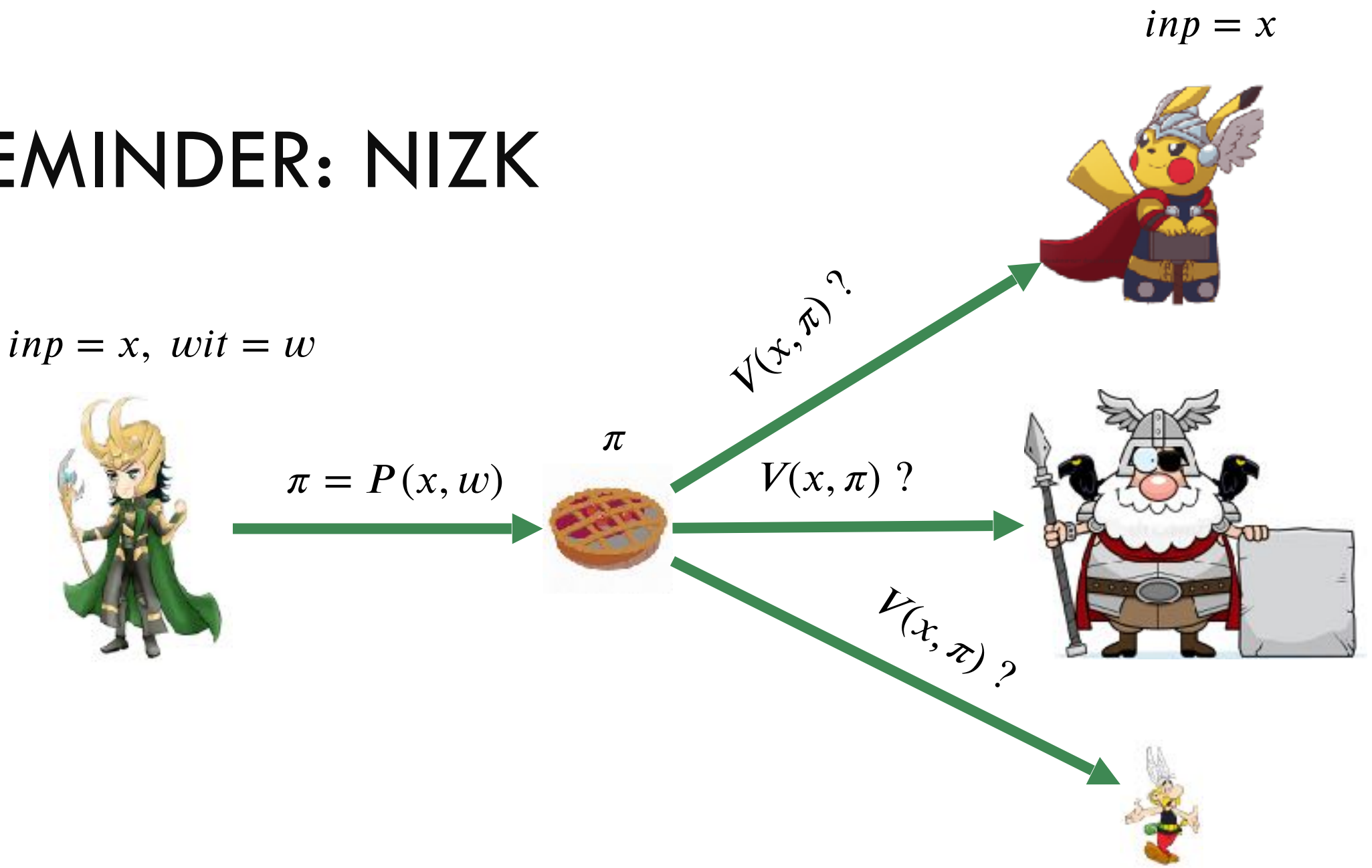

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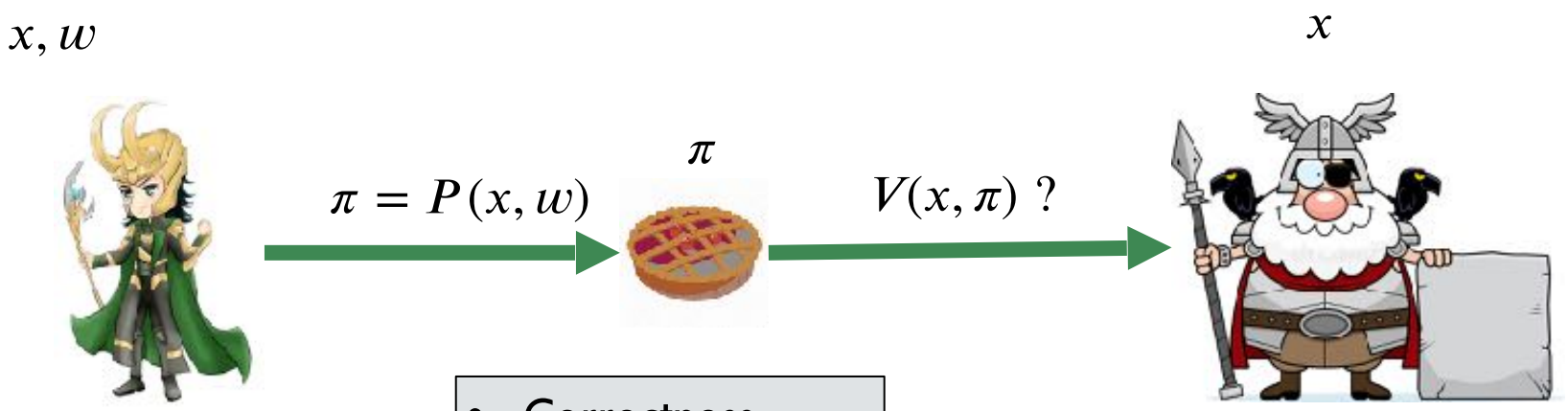

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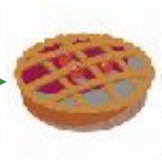

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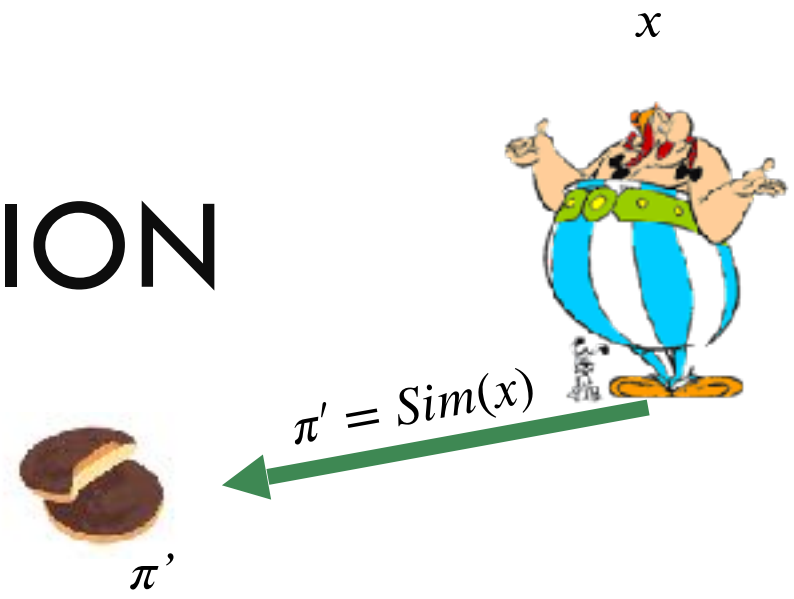

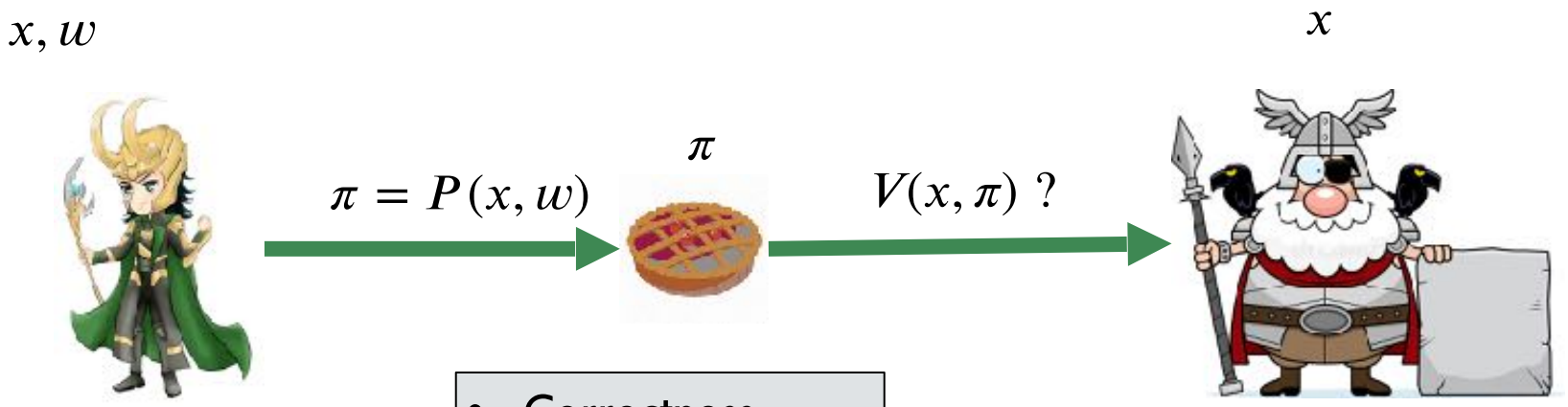

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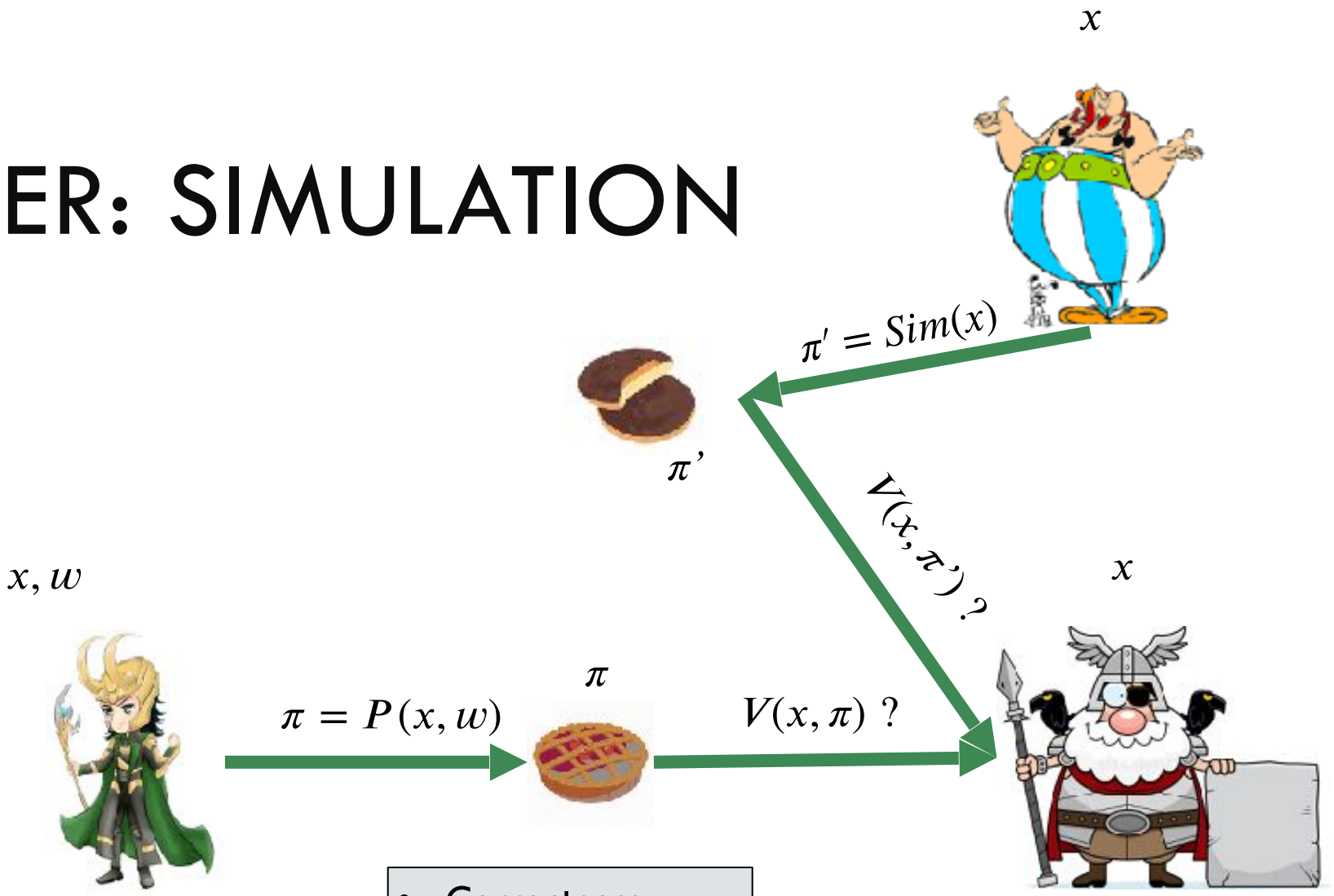

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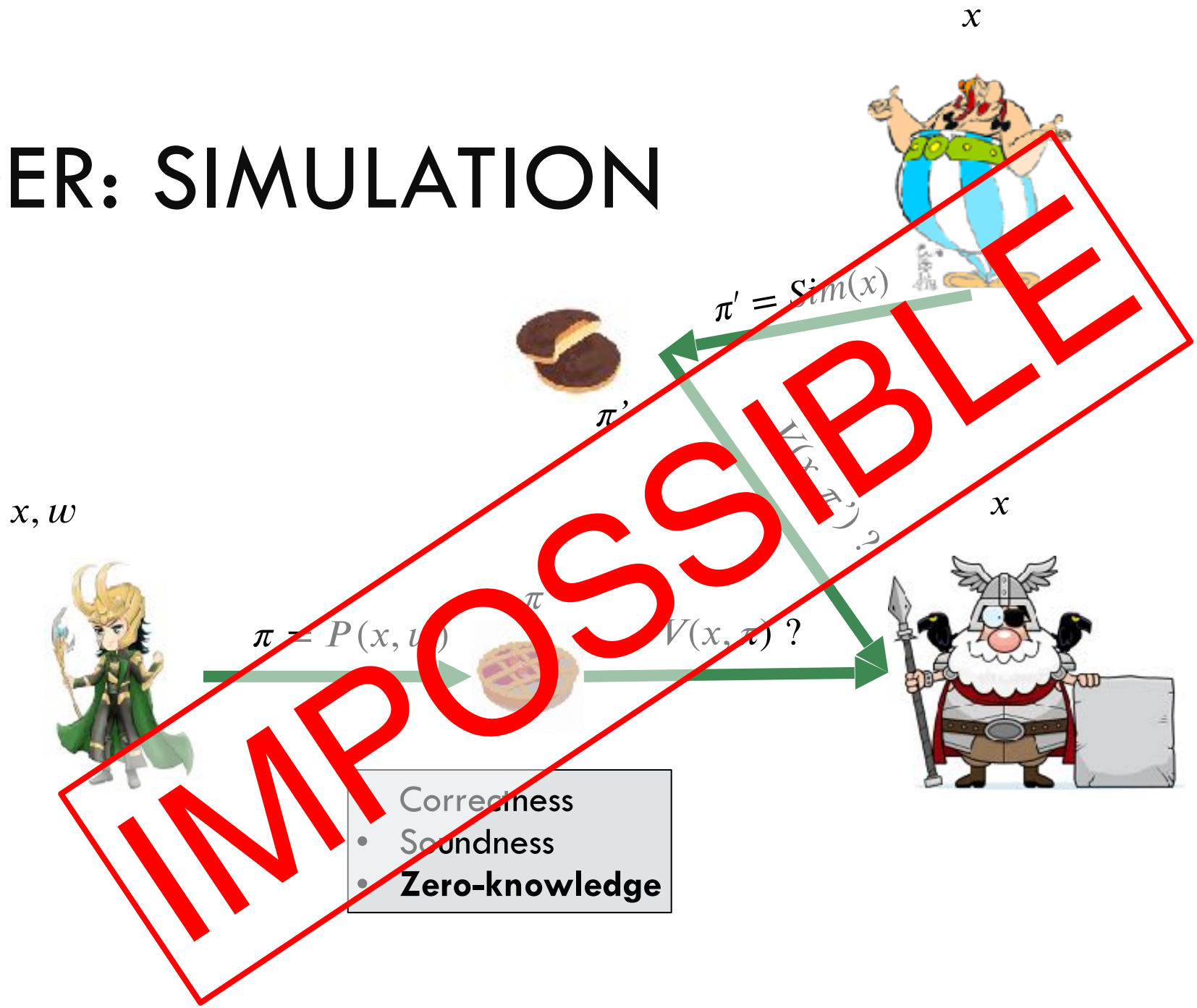

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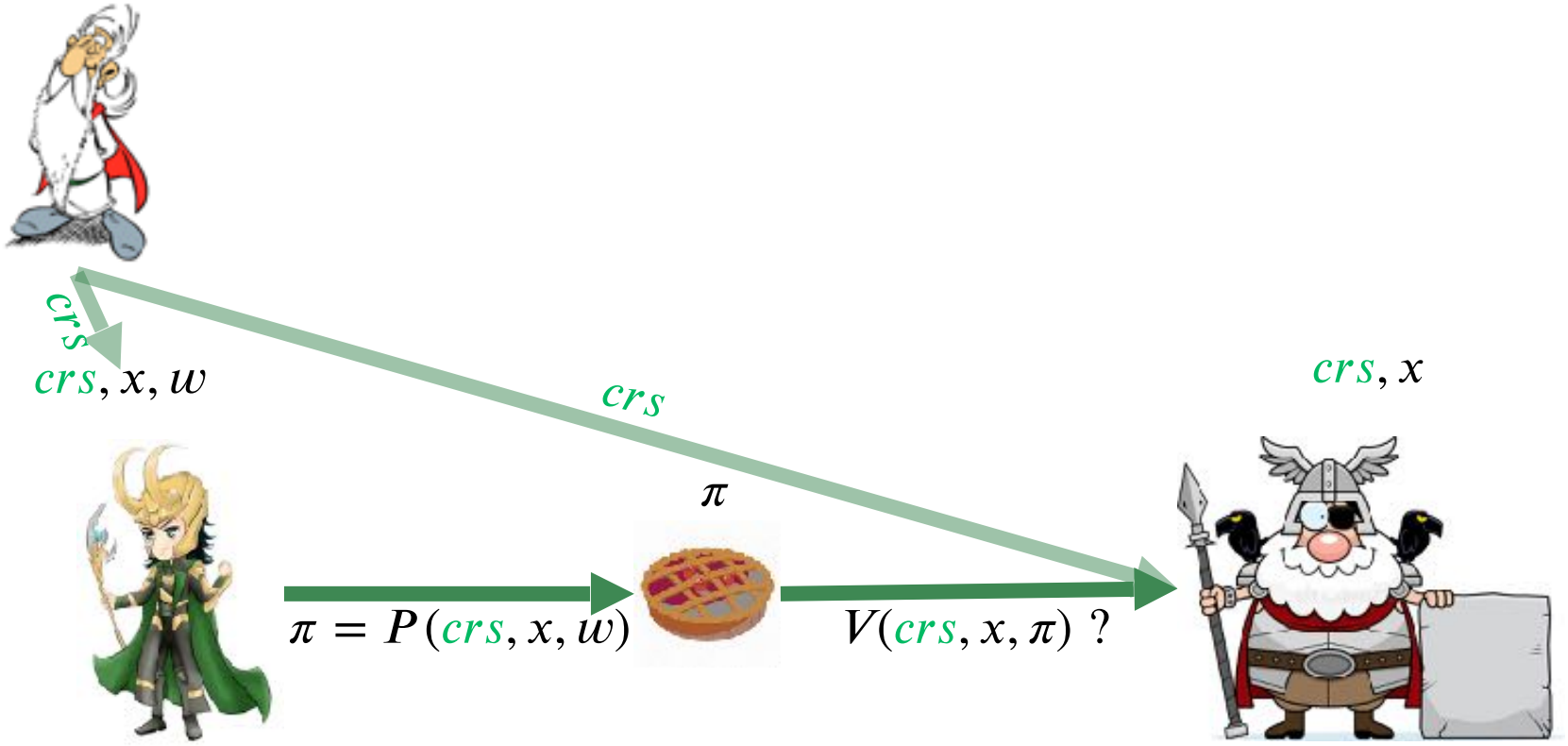

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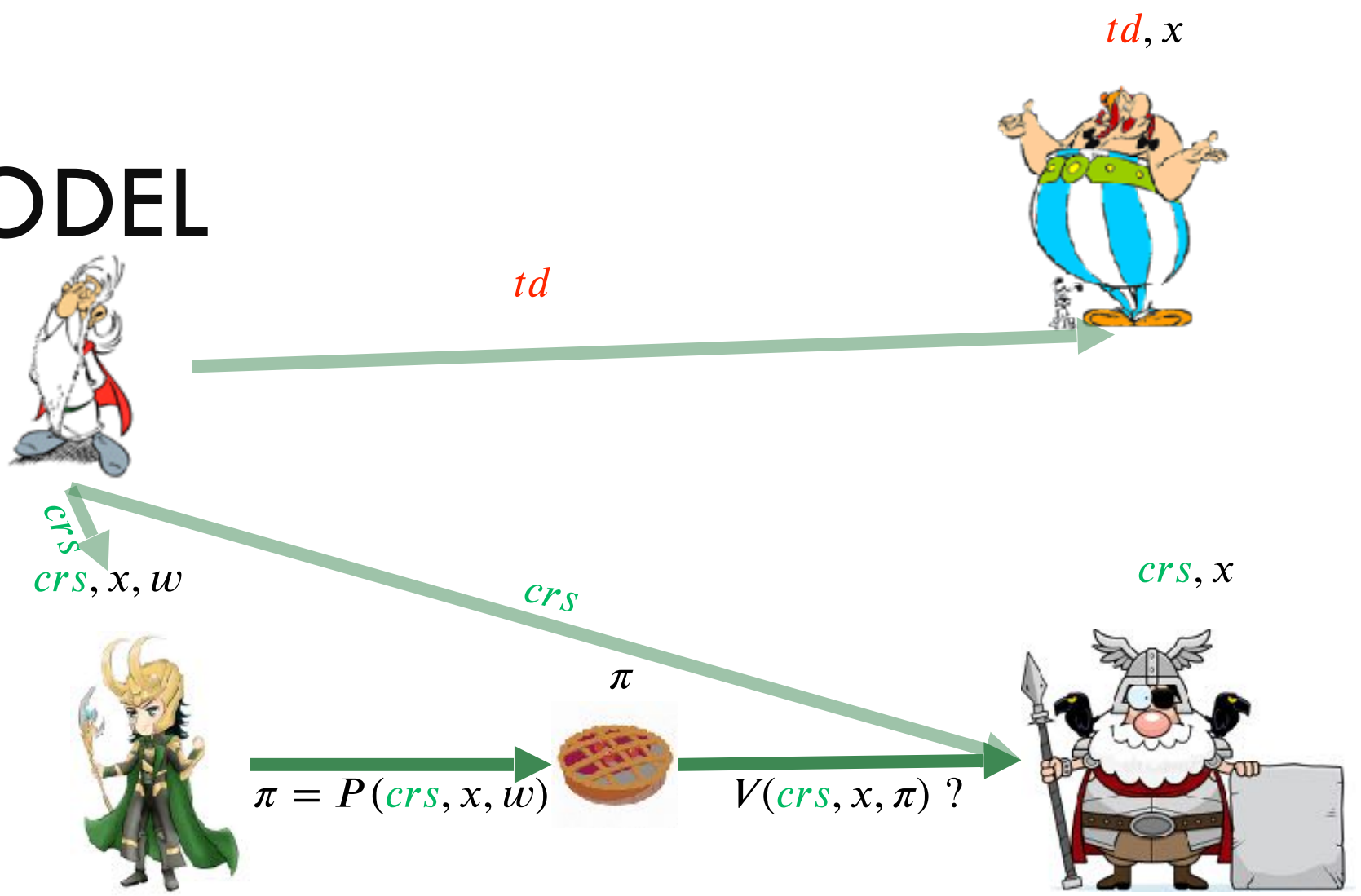

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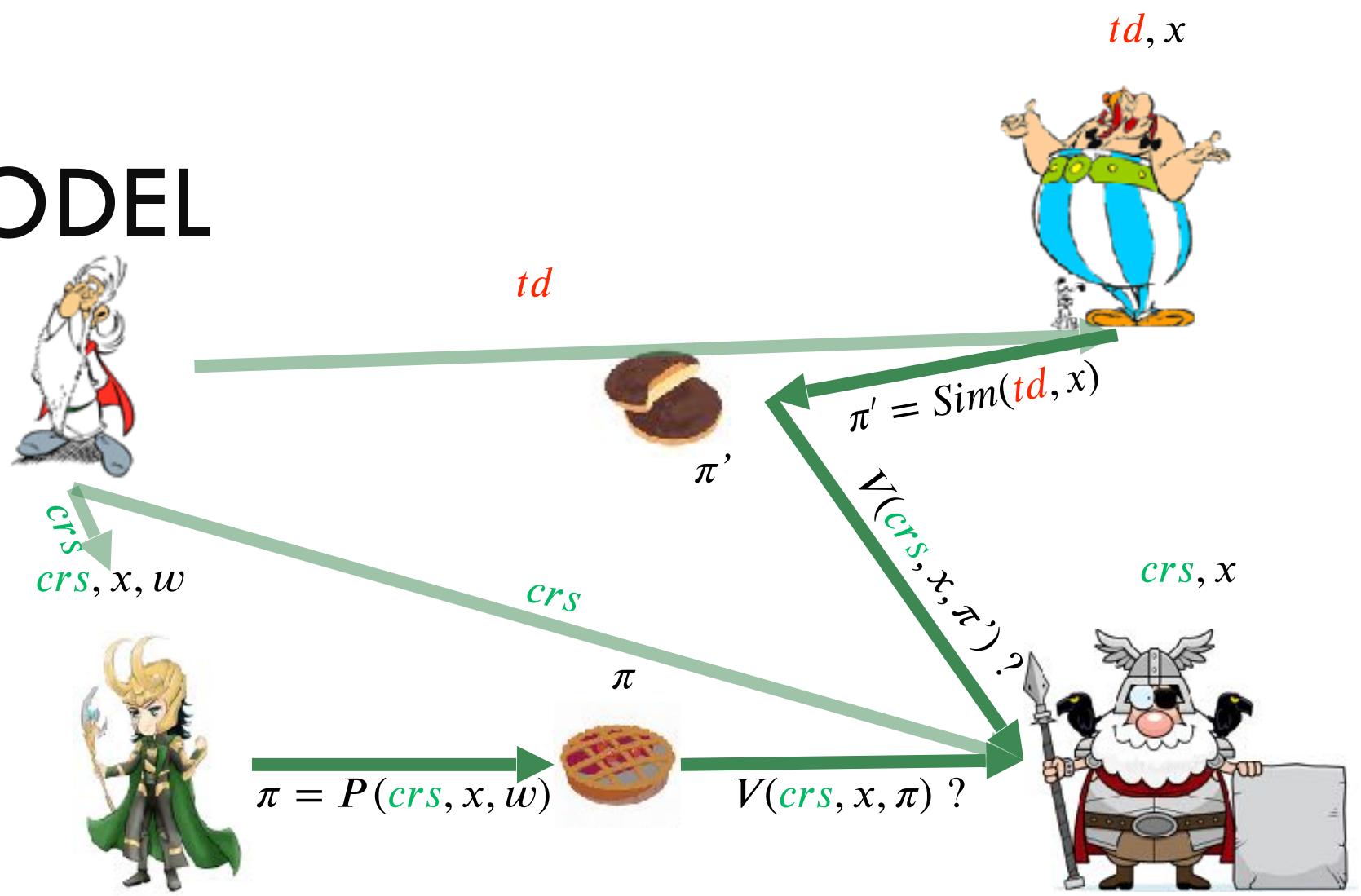

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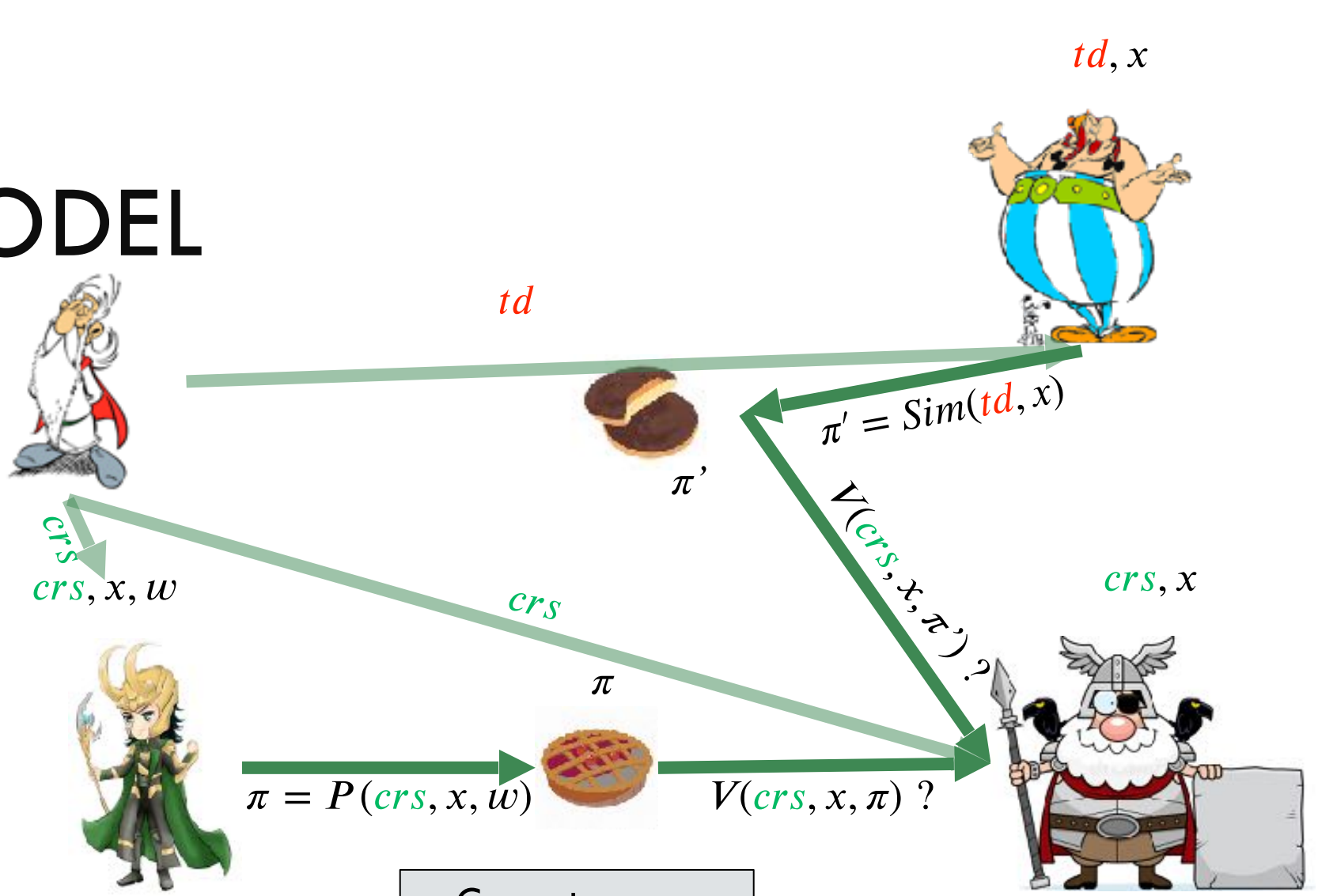

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  - building block of SNARK variations

QA-NIZK: DEFINITION

$lpar$

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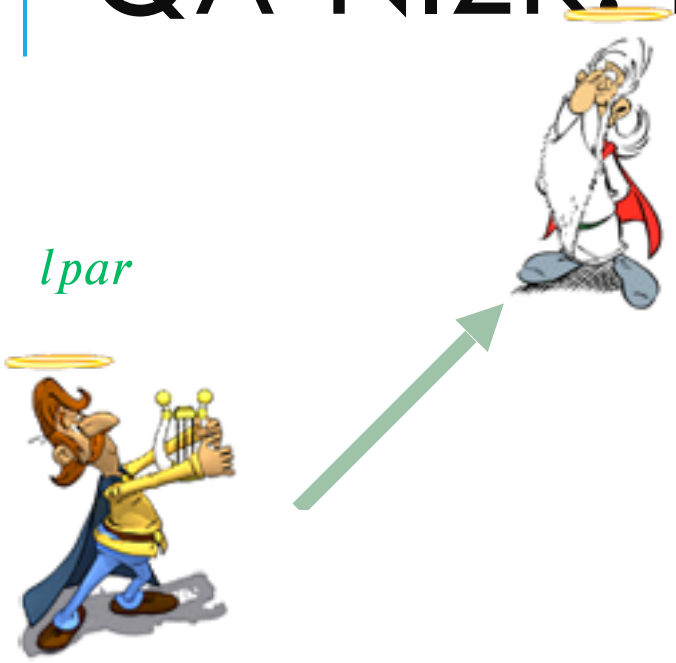

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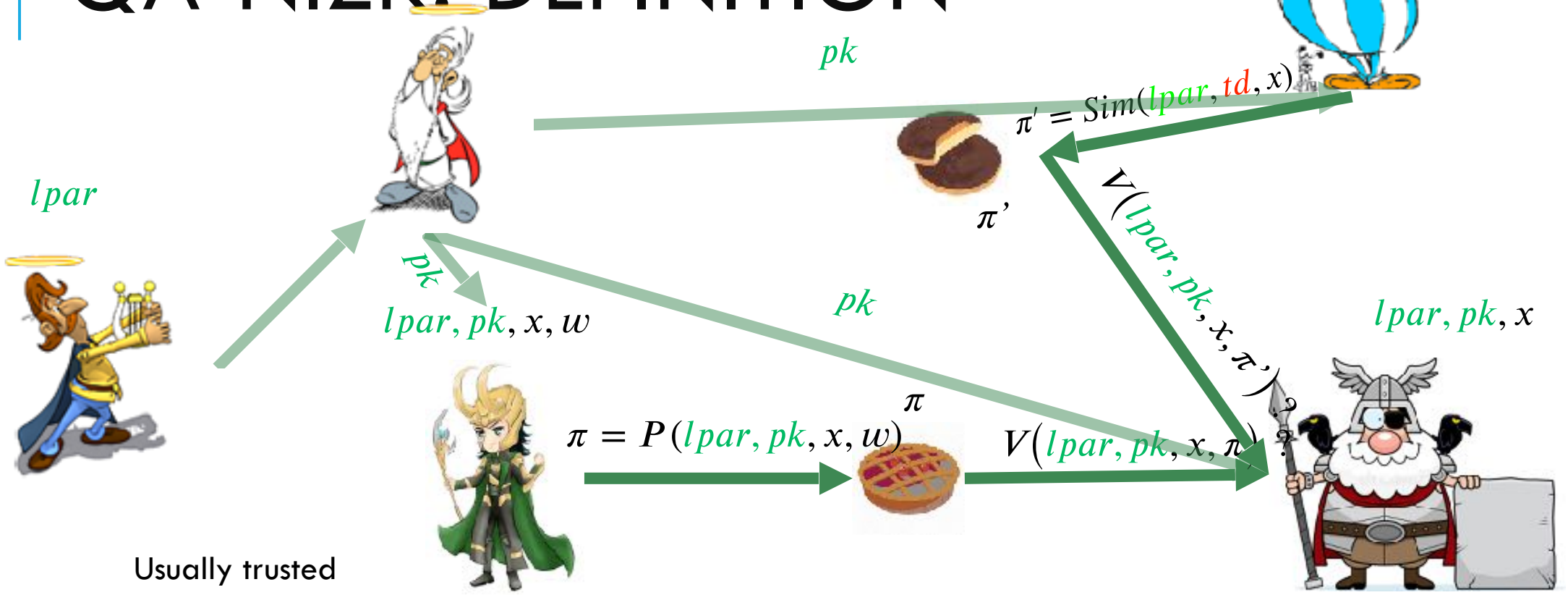

Usually trusted

quasi-adaptiveness enables to get improved efficiency
Subspace QA-NIZKs where $|\pi| = 1$ group element

EFFICIENT NIZKS (2): QUASI-ADAPTIVE NIZKS

Model	Year	Trust in lpar &/ pk	Efficiency	Assumptions
CRS Model	JR13	Everybody trusts both	Efficient for linear languages, less efficient for NP	Black-box
Sub-ZK (BPK model)	ALSZ20	V trusts pk generator	-..-	NBB for ZK (needed)
Updatable	This work	V trusts one pk updater	-..-	-..-

UPDATABLE SNARKS: IDEA

- **Non-updatable zk-SNARKs**
  - $sk = \mathbf{x}$ — a vector of uniformly random integers
  - $pk_C(\mathbf{x}) = \{[f(\mathbf{x})] : f \in S_C\}$ for a C**-dependent** function set S_C
- **Updatable SNARKs**
  - $pk_u(\mathbf{x}) := \{[f(\mathbf{x})] : f \in S_n\}$ for a universal set S_n
    - …universal for all circuits of size $\leq n$
  - verifiably specialize $pk_u(\mathbf{x}) \Rightarrow pk_C(\mathbf{x})$ to concrete circuit C

UPDATABLE SNARKS: DETAILS

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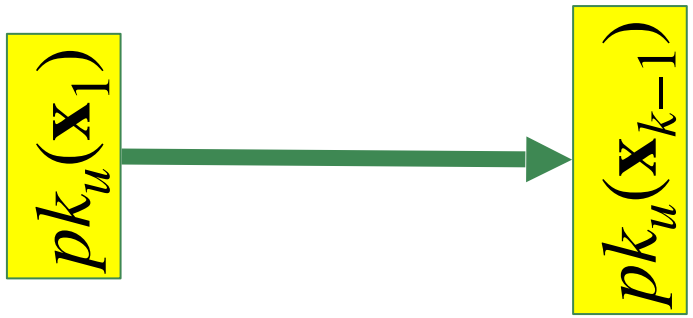

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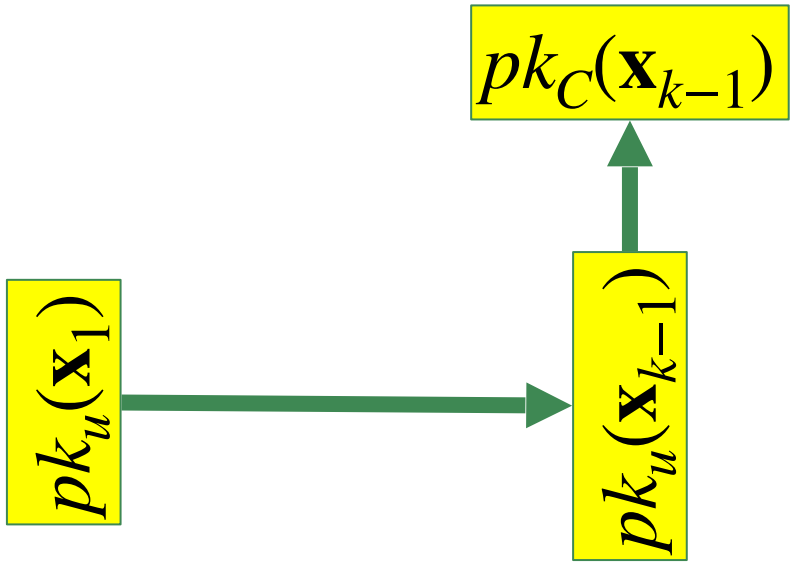

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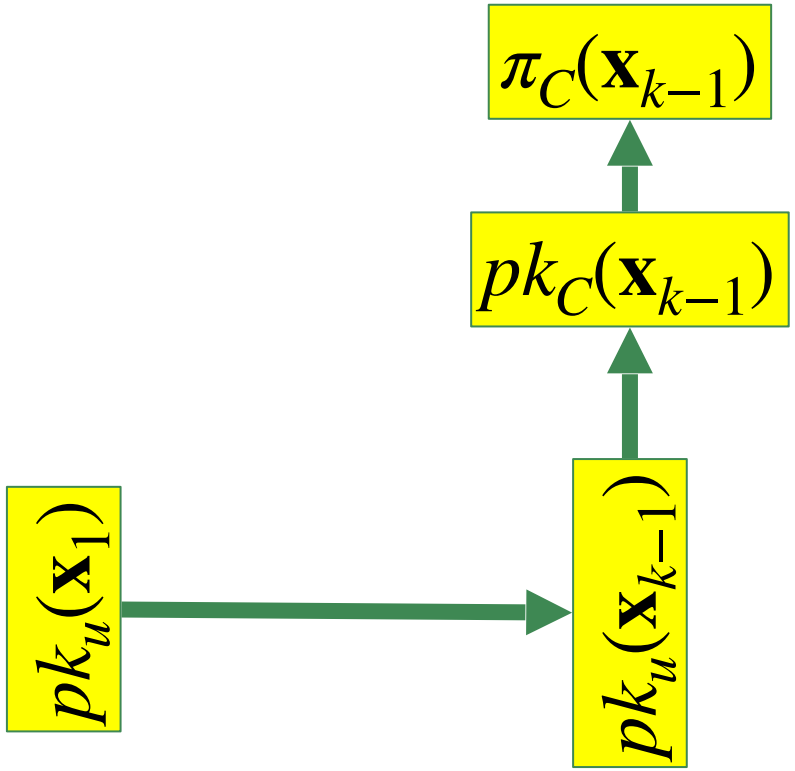

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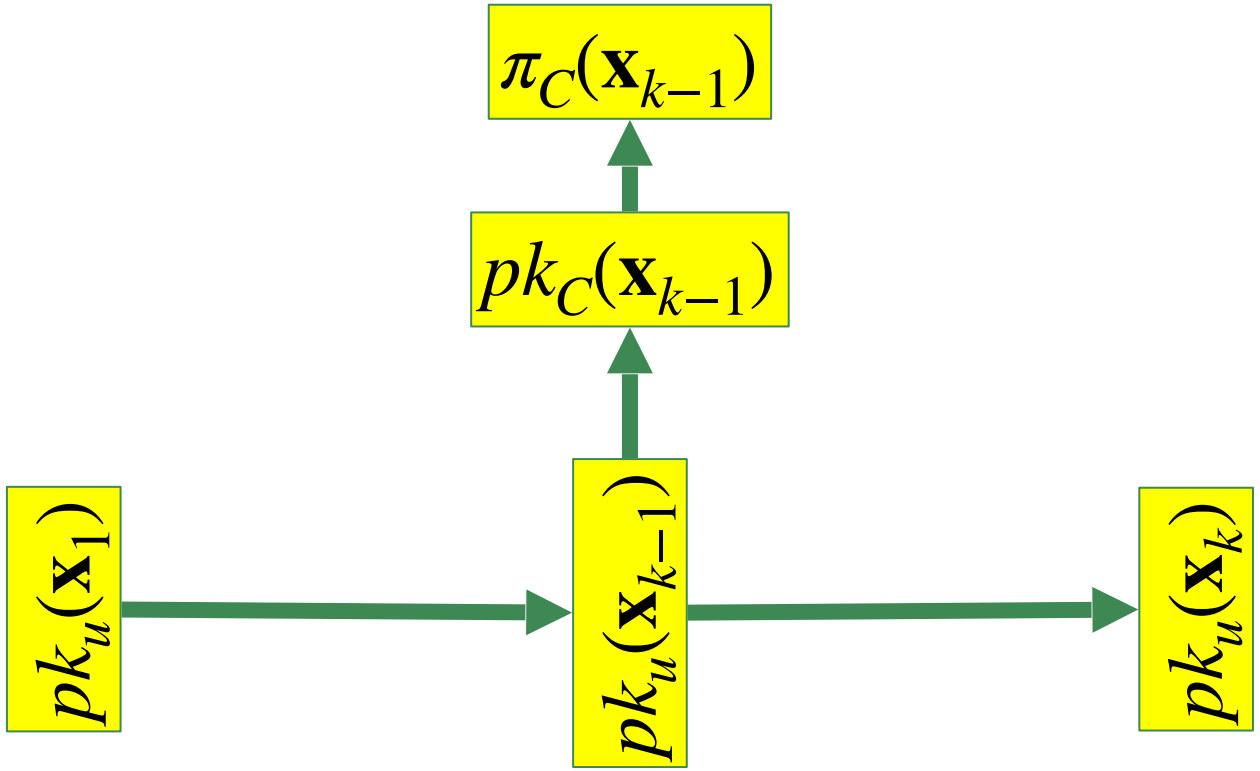

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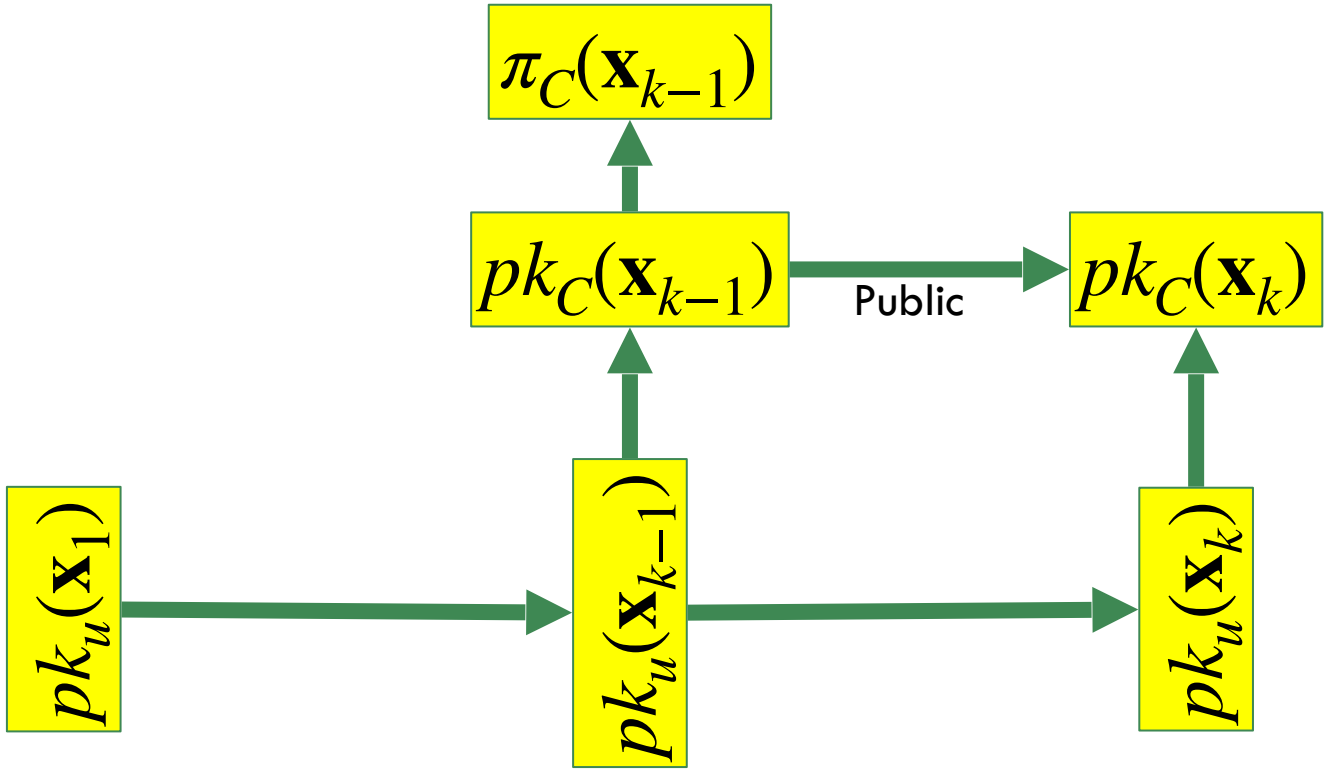

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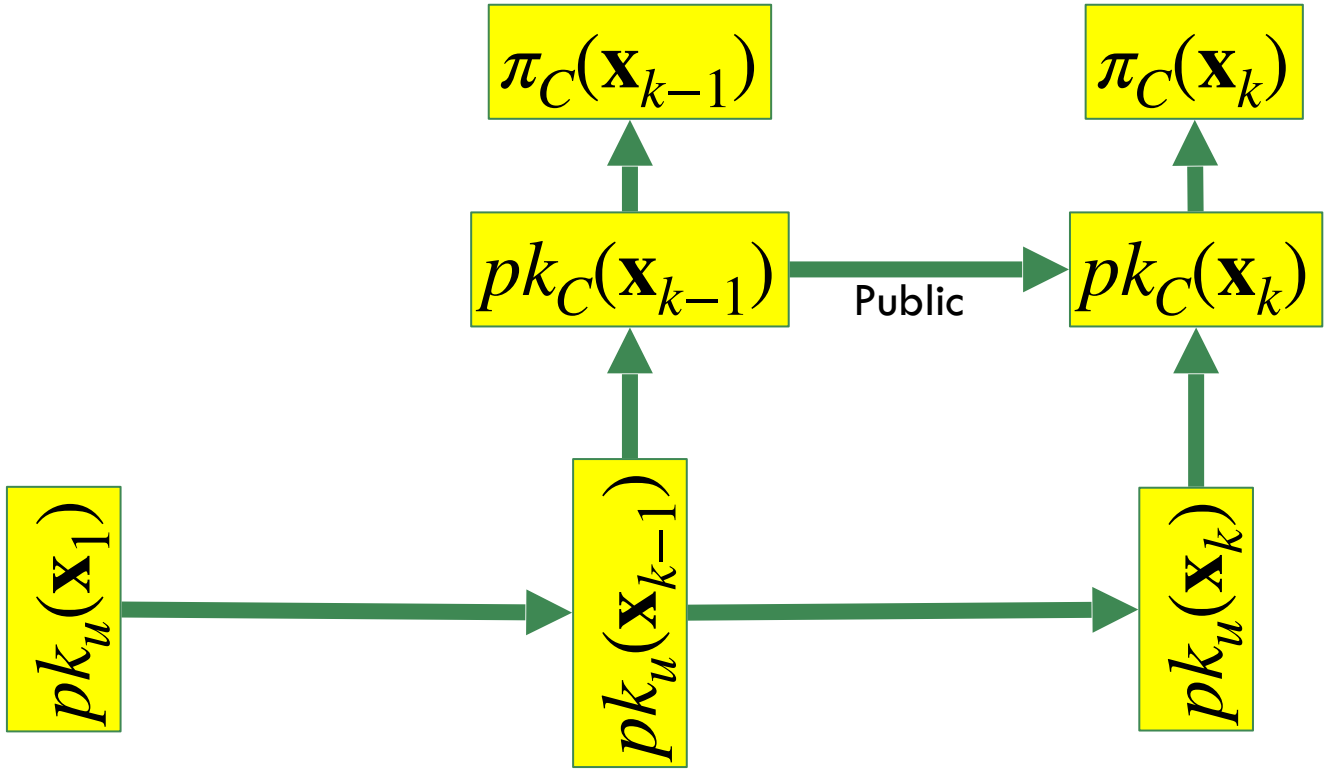

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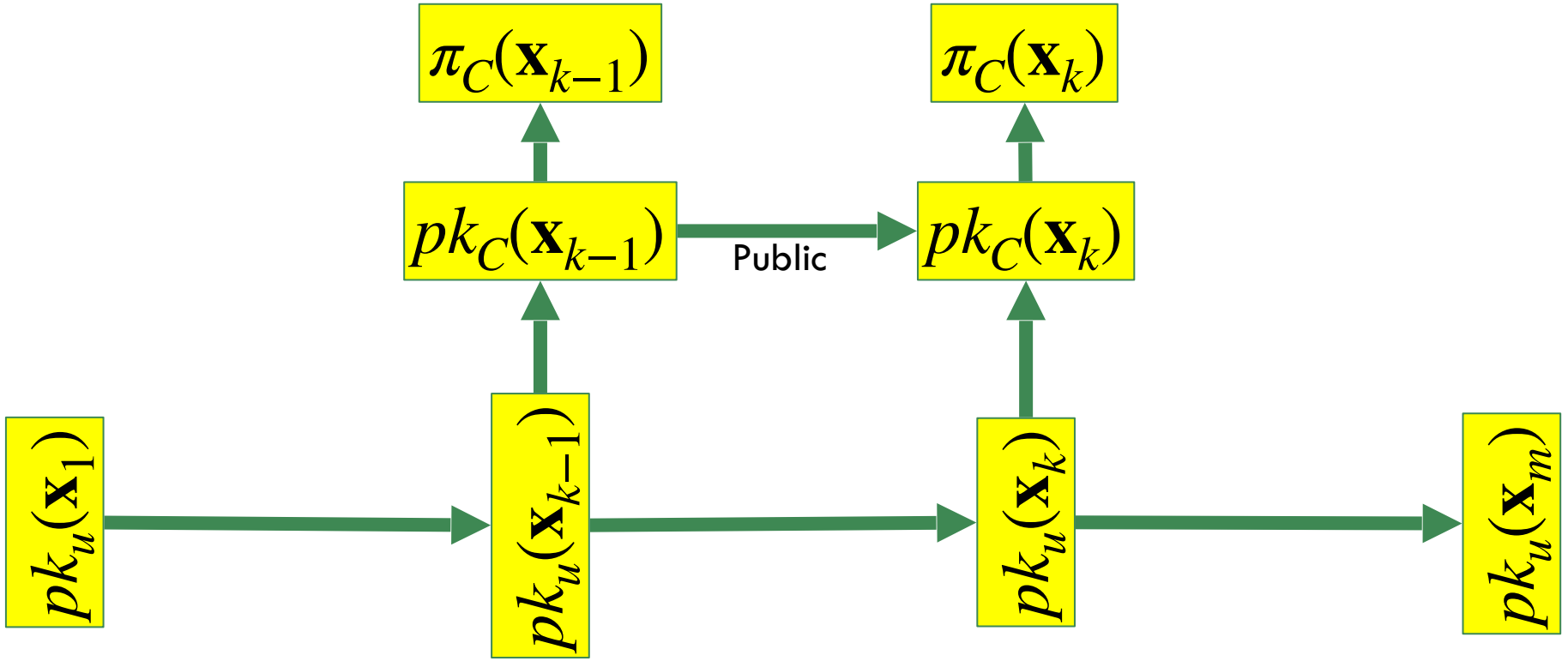

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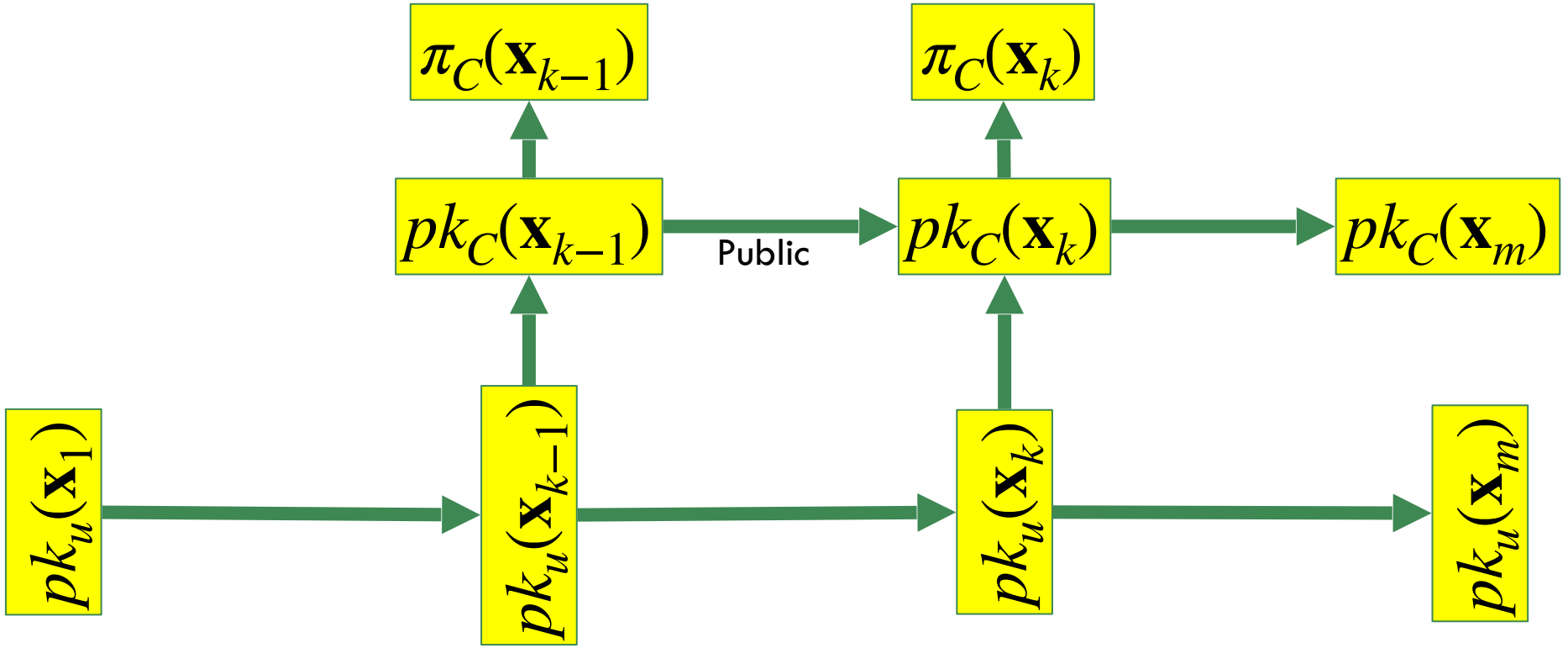

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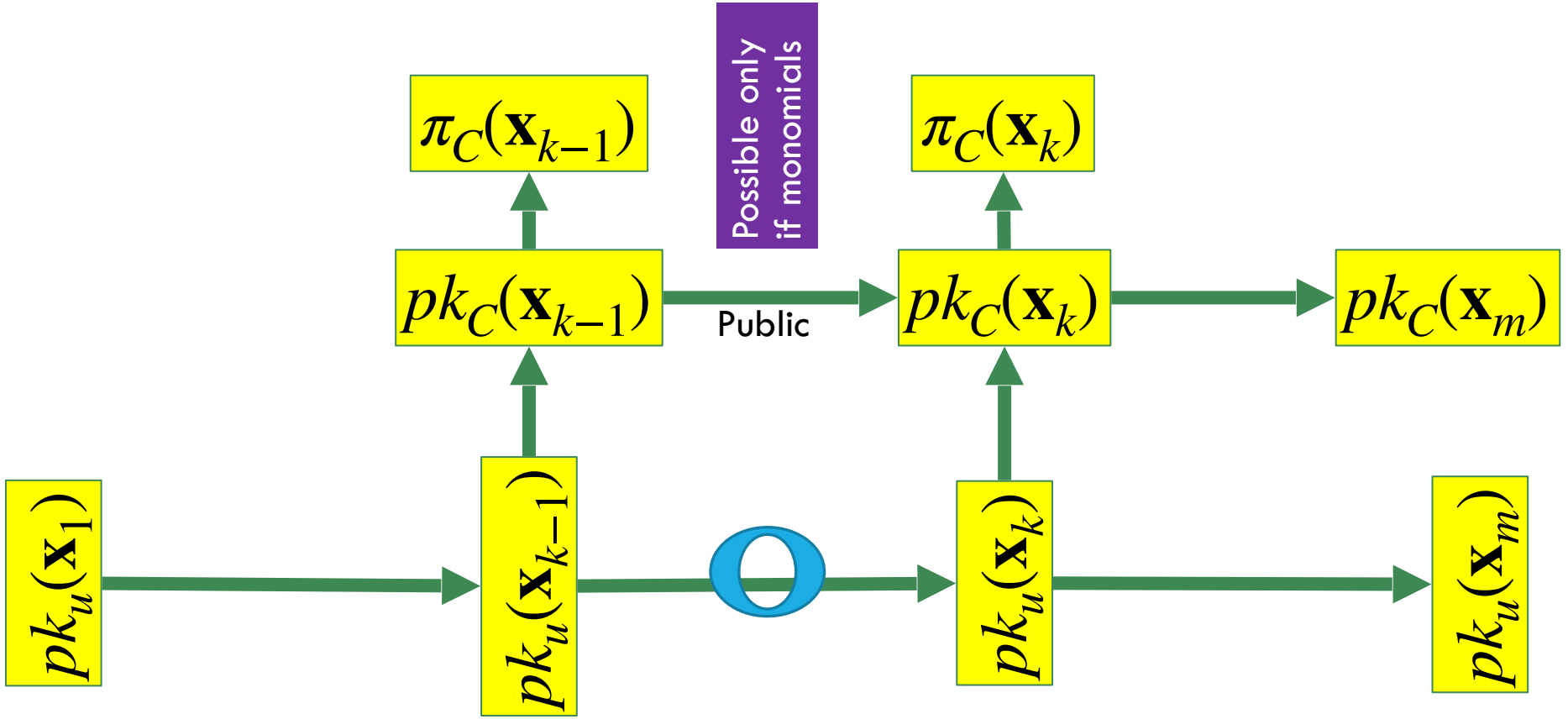

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  - Our observation: for this we need the distribution of sk belongs to a generalized ideal of a convolution semigroup ("nice distribution")

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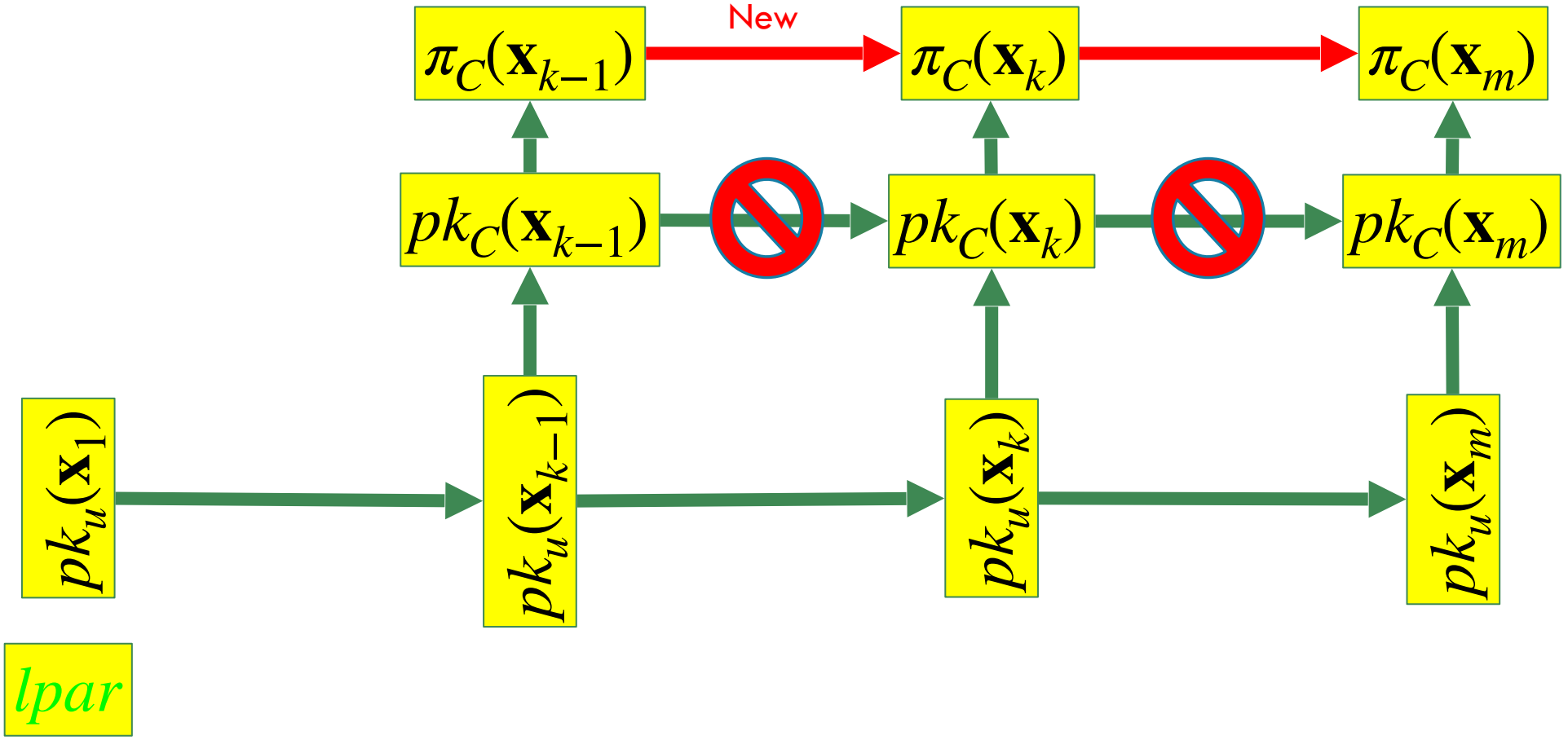
  - Update the rest of the pk separately in each application

K&A UPDATABLE QA-NIZKS



K&A UPDATABLE QA-NIZKS

Caveat: updated π is as sound as original π